

HIGGS BOSONS WITHIN AND BEYOND THE STANDARD MODEL

4th Graduate School in Physics at Colliders
June 29-July 3, 2009

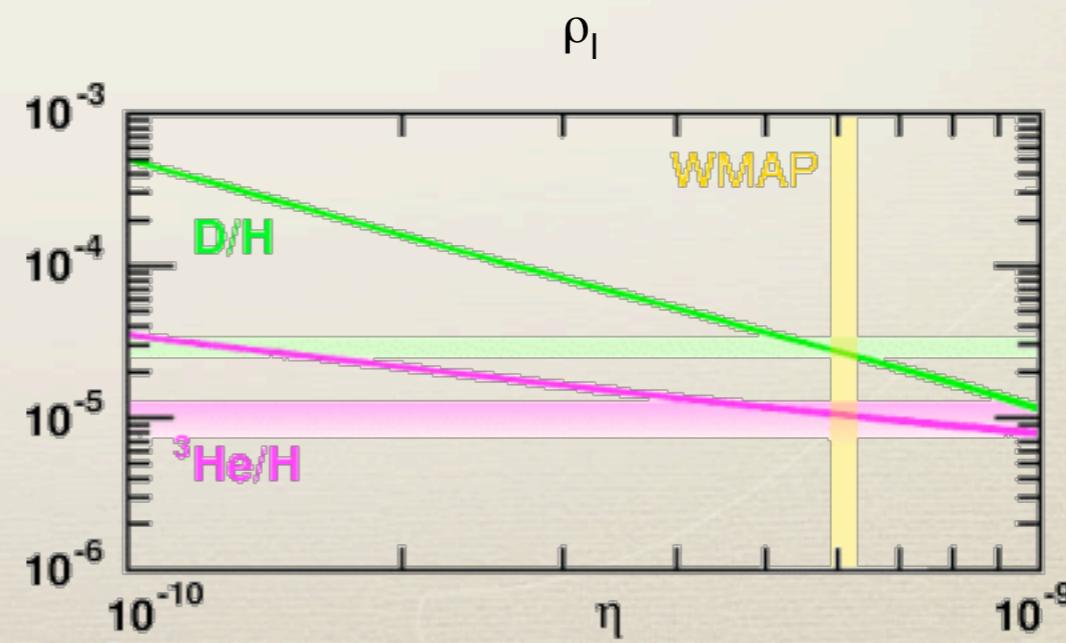
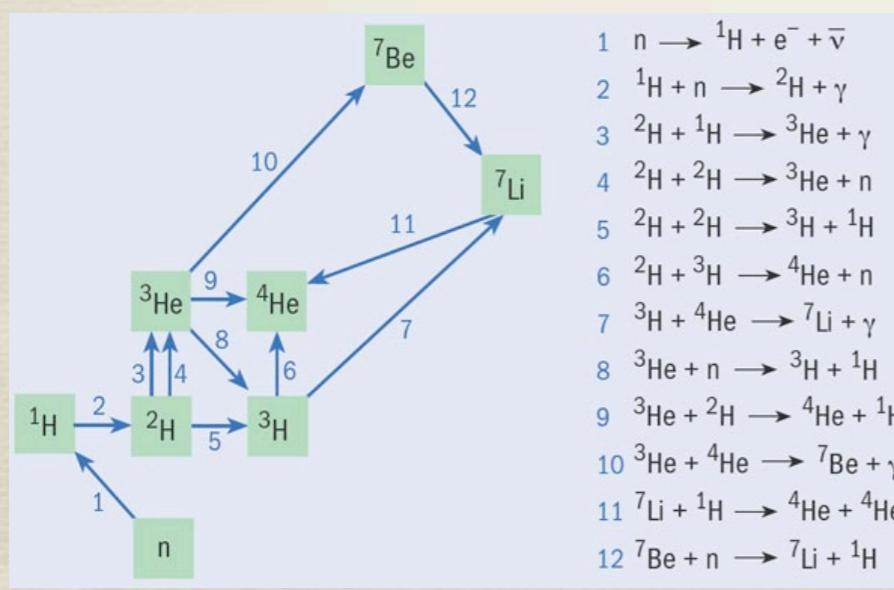
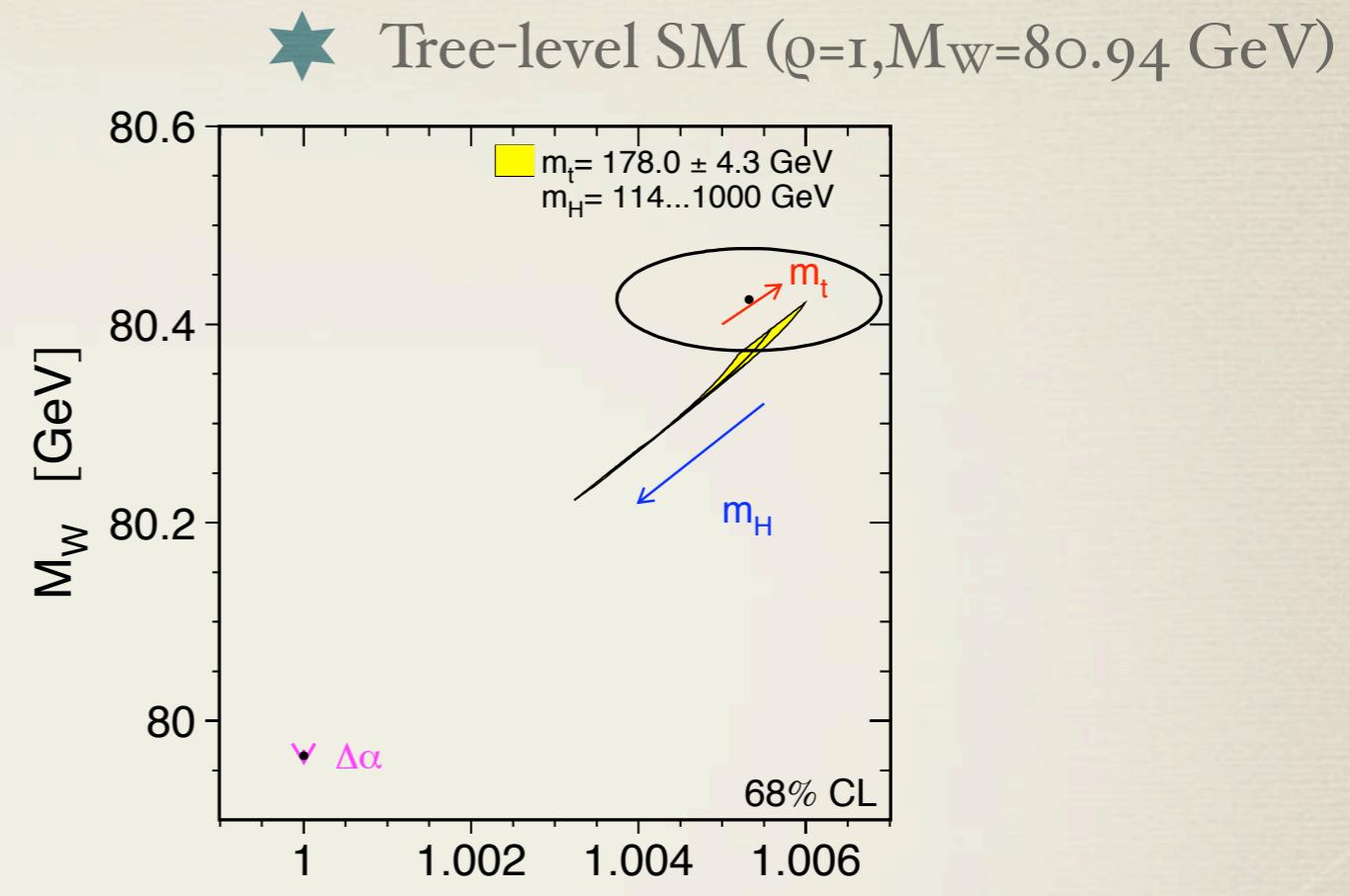
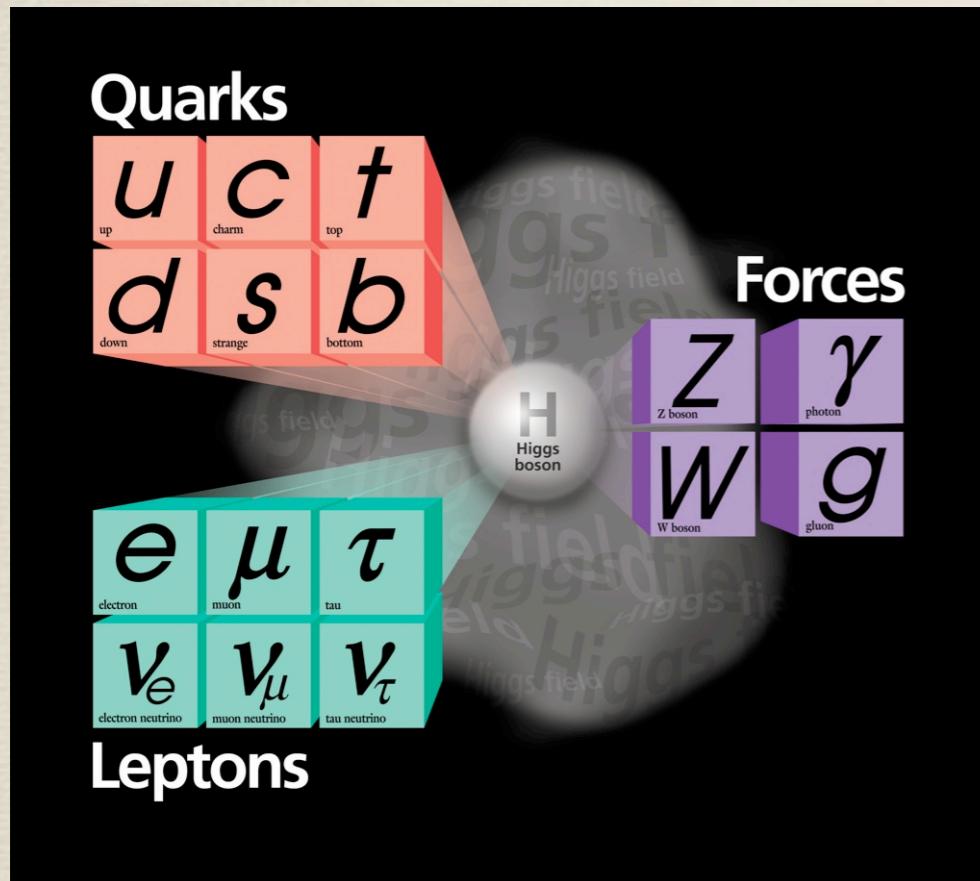
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Outline

- * Review of the SM and the Higgs mechanism
- * Constraining the Higgs: theoretical constraints and electroweak precision
- * A phenomenological profile: decays of the Higgs boson
- * Production mechanisms at e^+e^- and hadron colliders
- * A case study in QCD: gluon-fusion production
- * Searches at the Tevatron and the LHC
- * Measuring Higgs properties

Mostly SM, but will try to mention possible deviations

Success of the Standard Model



Building a gauge theory

- * Guiding principle in construction of SM is *gauge symmetry*
- * Pick a gauge group
- * Assign matter fields (fermions, scalars) to a *representation* of the gauge group, e.g., the *fundamental* N-component vector for SU(N)
- * To make the matter Lagrangian gauge invariant, replace $\partial_\mu \rightarrow D_\mu$

$$\mathcal{L} = -\frac{1}{2} \text{Tr} [F_{\mu\nu} F^{\mu\nu}] + \mathcal{L}_{\text{matter}} (\Psi, D_\mu \Psi)$$

gives Feynman rules
for gauge self-interactions

↑
governs gauge-matter
interactions

The Standard Model

- * Gauge group: $SU(3)_C \times SU(2)_L \times U(1)_Y$ (8 gluons; eventually photon, W^\pm, Z)
- * Three generations of fermionic matter

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} :$	3	2	1/6
$u_R :$	3	1	2/3
$d_R :$	3	1	-1/3
$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} :$	1	2	-1/2
$e_R :$	1	1	-1

- * Electric charge: $Q = T_3 + Y$

Problems with mass

- * The Lagrangian of the SM:

$$\begin{aligned} \mathcal{L}_{gauge+ferm} = & -\frac{1}{4} \overbrace{B_{\mu\nu} B^{\mu\nu}}^{U(1)_Y} - \frac{1}{4} \overbrace{W_{\mu\nu}^a W_a^{\mu\nu}}^{SU(2)_L} - \frac{1}{4} \overbrace{G_{\mu\nu}^a G_a^{\mu\nu}}^{SU(3)_C} \\ & + \underbrace{\sum_f i \bar{f} \not{D} f}_{f=Q_L, u_R, d_R, L_L, e_R} \end{aligned}$$

- * We know the W^\pm, Z bosons have mass, but this is not allowed by gauge symmetry

$$\mathcal{L}_{mass}^{SU(2)} = \frac{1}{2} m^2 W_\mu^a W_a^\mu \Rightarrow \Delta \mathcal{L}_{mass}^{SU(2)} \neq 0 \text{ under G.T.}$$

- * Similarly, fermion mass terms are not allowed by $SU(2)_L$ or $U(1)_Y$

$$\mathcal{L}_{mass}^{ferm} = -m \underbrace{[\bar{f}_R f_L + \bar{f}_L f_R]}$$

transforms as $SU(2)_L$ doublet, $\sum Y \neq 0$

Spontaneous symmetry breaking

- * The solution: Lagrangian is symmetric, ground state isn't \Rightarrow *spontaneous symmetry breaking*
- * Complex scalar transforming as $(1,2,1/2)$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$

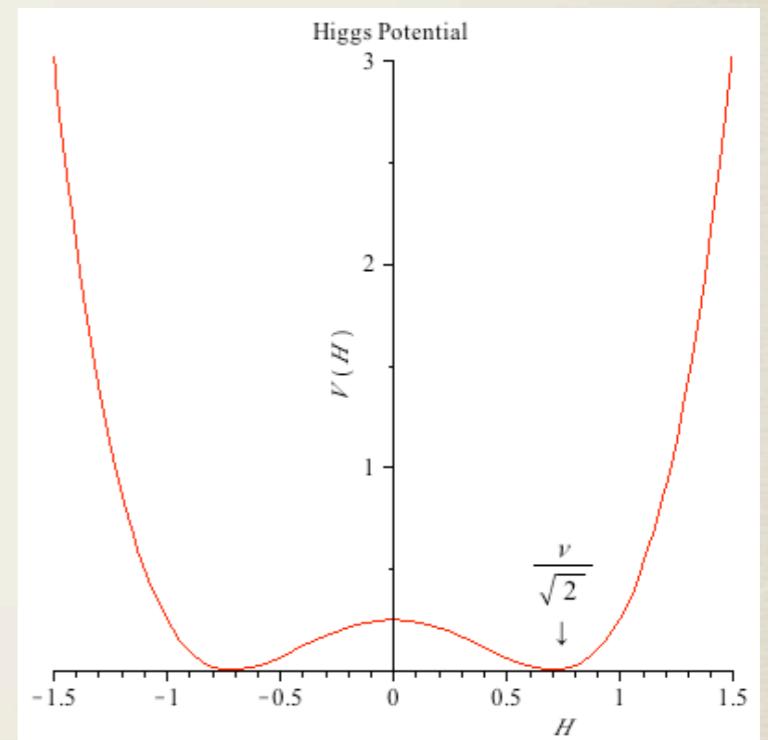
$$\mathcal{L}_{Higgs} = \overbrace{(D_\mu H)^\dagger D^\mu H - \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2}^{V(H)}$$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$D^\mu = \partial^\mu - igW_a^\mu \frac{\sigma^a}{2} - ig'B^\mu \frac{1}{2}$$

Vacuum expectation value: $\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

Expand around vev: $H = \begin{pmatrix} \phi^+ \\ \frac{v+h+i\chi}{\sqrt{2}} \end{pmatrix}$



$(\phi^+, \chi$ can be removed by G.T., set to zero)

The Higgs mechanism

- * Work out the kinetic part of Higgs Lagrangian

$$D_\mu H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2} \left[\frac{v+h}{\sqrt{2}} \right] \begin{pmatrix} \sqrt{2}g W_\mu^+ \\ \sqrt{g^2 + g'^2} Z_\mu \end{pmatrix}$$

$$(D^\mu H)^\dagger D_\mu H = \frac{1}{2} \partial_\mu h \partial^\mu h + \left(1 + \frac{h}{v} \right)^2 \left(\underbrace{\frac{g^2 v^2}{4} W^\mu_+ W_\mu^-}_{M_W^2} + \frac{1}{2} \underbrace{\frac{(g^2 + g'^2)v^2}{4} Z_\mu Z^\mu}_{M_Z^2} \right)$$

$$Z_\mu = c_W W_\mu^3 - s_W B_\mu, A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}$$

$$c_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad s_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

- * W^\pm, Z acquire mass by “eating” φ^+, χ

True for arbitrary # of doublets

Prediction: $\rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1$
 (tree-level; more later)

Fermion masses

- * Yukawa interactions with Higgs doublets give fermions mass

$$\begin{aligned}\mathcal{L}_{Yuk} &= -\lambda_d \bar{Q}_L H d_R - \lambda_u \bar{Q}_L (i\sigma_2 H^*) u_R - \lambda_e \bar{L}_L H e_R + \text{h.c.} \\ &\Rightarrow -\left(1 + \frac{h}{v}\right) \sum_{f=u,d,e} m_f \bar{f} f \quad \text{with} \quad m_f = \frac{\lambda_f v}{\sqrt{2}}\end{aligned}$$

(matrix in generation space, implicitly diagonalized at price of V_{CKM} in charged currents)

- * Sum of all pieces so far give the SM Lagrangian:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge+ferm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$

- * The single Higgs doublet is just the simplest way to break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$; EWSB could be more intricate.
But this is the benchmark to compare other theories against.

Feynman rules

- * Work out the experimental predictions with Feynman rules:

$$\begin{array}{c} h \dashrightarrow f \\ \text{---} \nearrow \searrow \\ f \quad f \end{array} = -i \frac{m_f}{v}$$

$$\begin{array}{c} h \dashrightarrow \\ \text{---} \nearrow \swarrow \\ h \quad w \end{array} = 2i \frac{M_W^2}{v^2} g_{\mu\nu}$$

$$\begin{array}{c} h \dashrightarrow \\ \text{---} \nearrow \swarrow \\ w \quad w \end{array} = 2i \frac{M_W^2}{v} g_{\mu\nu}$$

$$\begin{array}{c} h \dashrightarrow \\ \text{---} \nearrow \swarrow \\ h \quad z \end{array} = 2i \frac{M_Z^2}{v^2} g_{\mu\nu}$$

$$\begin{array}{c} h \dashrightarrow \\ \text{---} \nearrow \swarrow \\ z \quad z \end{array} = 2i \frac{M_Z^2}{v} g_{\mu\nu}$$

From muon decay,
 $v^2=I/(G_F\sqrt{2}) \Rightarrow v \approx 246 \text{ GeV}$

- * Only scalars with vevs have linear HVV couplings

Test the consequences of the Higgs mechanism

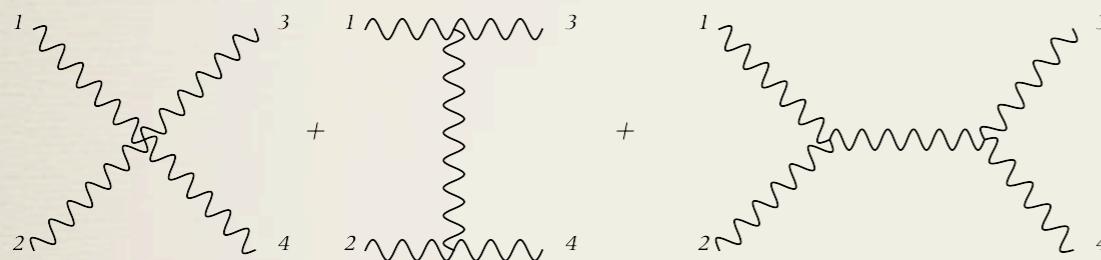
Theoretical constraints

- * Unitarity of S-matrix:

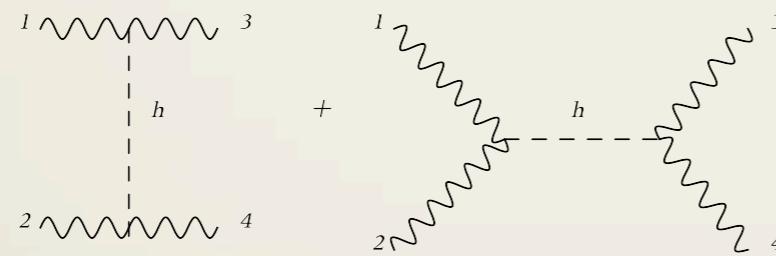
$$a_l = \frac{1}{32\pi} \int_{-1}^1 dc_\theta \underbrace{P_l(c_\theta)}_{\substack{\text{Legendre poly.} \\ \text{amplitude}}} \mathcal{M}$$

$$S^\dagger S = 1 \Rightarrow |\text{Re}(a_l)| < 1/2$$

Longitudinal modes: $\epsilon_L = (|p_l|/M, 0, 0, E/M)$



+



$$a_0(W_L W_L \rightarrow W_L W_L) \rightarrow -\frac{s}{32\pi v^2}$$

$$a_0(W_L W_L \rightarrow W_L W_L) \rightarrow -\frac{M_H^2}{8\pi v^2}$$

- * Probability not conserved without Higgs; with, $M_H < 900$ GeV (perturbative argument)

More in Christophe's lectures

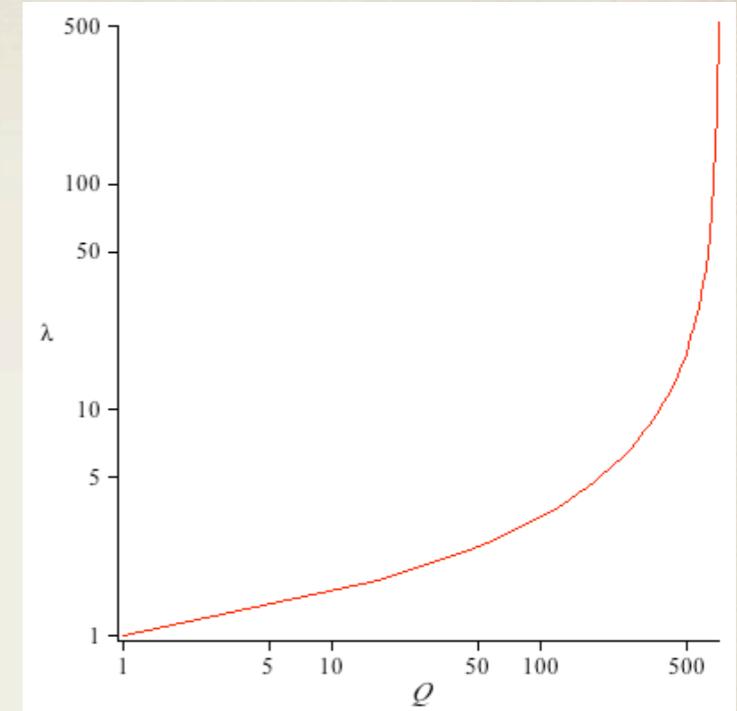
Theoretical constraints

- * Landau pole of λh^4 coupling

$$\lambda(Q) = \frac{M_H^2}{2v^2} \frac{1}{1 - \frac{3}{4\pi^2} \frac{M_H^2}{v^2} \ln \frac{Q}{v}} \quad (\text{large } \lambda \text{ limit})$$

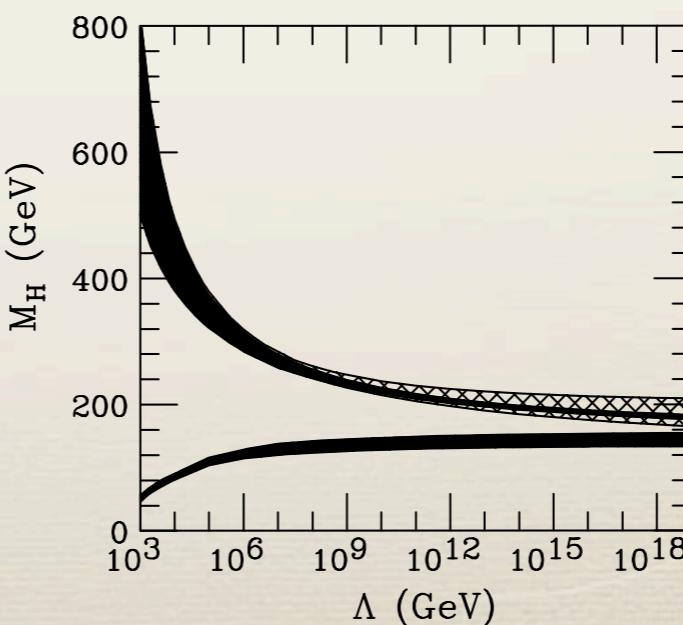
Breaks down at some Q

For validity up to $Q=\Lambda$ ($\lambda < \infty$),
upper bound on M_H



- * Shape of Higgs potential: $\lambda > 0 \Rightarrow$ lower bound on M_H

Validity of SM to high scales
restricts allowed M_H



More in Christophe's
lectures

Electroweak precision

- * Can experimentally probe properties of the Higgs directly (try to produce at a collider) or indirectly (through quantum effects)
- * LEP+SLC: millions of $e^+e^- \rightarrow Z \rightarrow ff$, high-precision measurements of SM electroweak parameters \Rightarrow effect of Higgs?
- * Study one-loop predictions of SM
- * Basic idea in renormalizable theory: fix most precisely known quantities, calculate others in terms of them

$$G_F = 1.166367(5) \times 10^{-5} \text{ GeV}^{-2} \text{ (muon decay)}$$

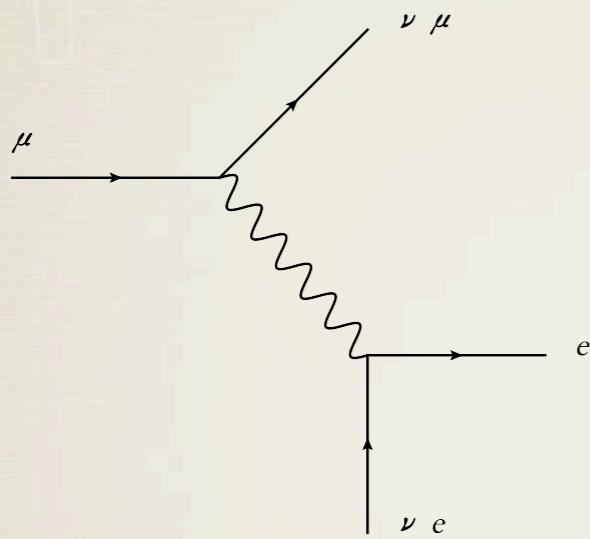
$$\alpha^{-1} = 137.035999679(94) \text{ (low-energy experiments)}$$

$$M_Z = 91.1875(21) \text{ (LEP)}$$

- * We'll work out prediction for M_W

Muon decay

- * Muon-decay at tree-level: (technical point: QED corrections accounted for in Fermi theory and included in G_F extraction)

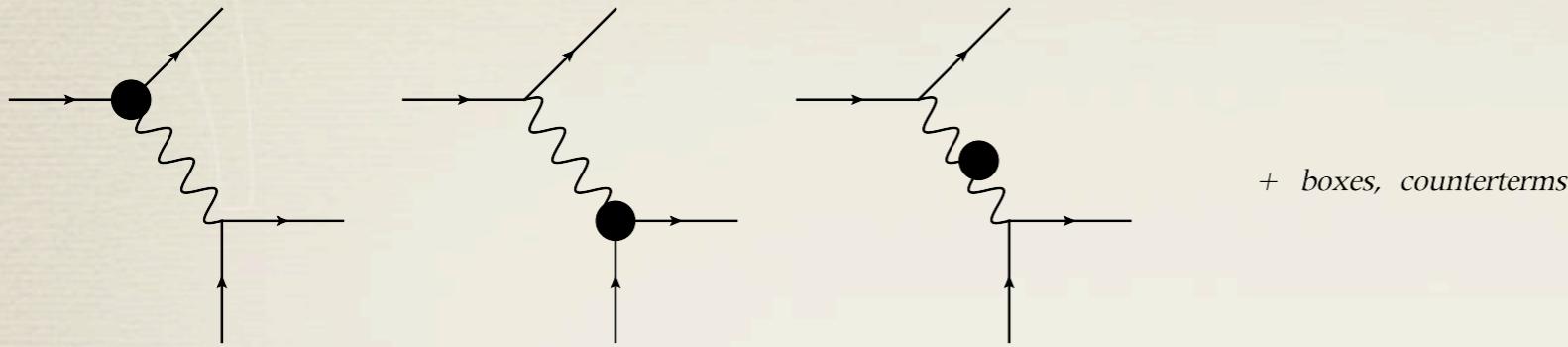


$$\begin{aligned}
 \frac{G_F}{\sqrt{2}} &= \frac{e^2}{8M_W^2 s_W^2} \quad (m_{e,\mu} = 0) \\
 s_W^2 &= 1 - \frac{M_W^2}{M_Z^2} \quad (\text{on-shell scheme}) \\
 \Rightarrow \frac{G_F}{\sqrt{2}} &= \frac{\pi\alpha}{2M_W^2 (1 - M_W^2/M_Z^2)} \\
 \Rightarrow M_W^2 &= \frac{M_Z^2}{2} \left\{ 1 + \left[1 - \frac{2\sqrt{2}\pi\alpha}{G_F M_Z^2} \right]^{1/2} \right\} \\
 &\approx 80.94 \text{ GeV} \Rightarrow \text{experiment gets } 80.4 \text{ GeV!}
 \end{aligned}$$

- * Keep only leading corrections (m_t , M_H , running of α ; others defined as ‘small’)

$$\begin{aligned}
 \frac{G_F}{\sqrt{2}} &= \frac{e^2}{8M_W^2 s_W^2} (1 + \Delta r) \\
 \Rightarrow M_W^2 &= \frac{M_Z^2}{2} \left\{ 1 + \left[1 - \frac{2\sqrt{2}\pi\alpha (1 + \Delta r)}{G_F M_Z^2} \right]^{1/2} \right\}
 \end{aligned}$$

Muon-decay at one loop



No vertex, box can depend
on m_t, M_H ($m_{e,\mu} \approx 0$) \Rightarrow only
self-energy, counterterms

$$\begin{aligned} e_0^2 &= e^2 - \delta e^2 \\ M_{W0}^2 &= M_W^2 + \delta M_W^2 \\ M_{Z0}^2 &= M_Z^2 + \delta M_Z^2 \\ s_{W0}^2 &= 1 - \frac{M_{W0}^2}{M_{Z0}^2} \end{aligned}$$

$$\begin{aligned} \frac{e_0^2}{s_{W0}^2 M_{W0}^2} &= \frac{e^2}{s_W^2 M_W^2} \left\{ 1 - \frac{\delta e^2}{e^2} - \frac{c_W^2}{s_W^2} \left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right] - \frac{\delta M_W^2}{M_W^2} \right\} \\ \Delta r_2 &= -\frac{\delta e^2}{e^2} - \frac{c_W^2}{s_W^2} \left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right] - \frac{\delta M_W^2}{M_W^2} \end{aligned}$$

$$\begin{aligned} \text{---} &= \frac{ig_{\mu\rho}}{M_W^2} [i\Pi_{WW}(0)] \frac{ig_{\rho\nu}}{M_W^2} \\ \Rightarrow & \frac{ig_{\mu\nu}}{M_W^2} \left[1 - \frac{\Pi_{WW}(0)}{M_W^2} \right] \\ \Rightarrow & \Delta r_1 = -\frac{\Pi_{WW}(0)}{M_W^2} \end{aligned}$$

$\boxed{\Delta r = \Delta r_1 + \Delta r_2 + \Delta r_{rem}}$

Muon decay at one-loop

on-shell mass renormalization :

$$\delta M_V^2 = \Pi_{VV}(M_V^2)$$

$$\Pi_{VV}(M_V^2) = \Pi_{VV}(0) + \underbrace{\dots}_{small}$$

charge renormalization :

$$\delta e^2/e^2 = \Pi_{\gamma\gamma}(0)$$

$$\begin{aligned} \Pi_{\gamma\gamma}(0) &= -[\Pi_{VV}(M_Z^2) - \Pi_{VV}(0)] + \underbrace{\Pi_{VV}(M_Z^2)}_{small} & [\Pi_{\gamma\gamma}(M_Z^2) - \Pi_{\gamma\gamma}(0)] &\sim \ln \frac{M_Z^2}{m_f^2} \\ &\approx -\frac{\alpha(M_Z^2) - \alpha(0)}{\alpha(0)} \equiv -\Delta\alpha \quad (\text{non-perturbative; light quarks}) \end{aligned}$$

Combine all terms to obtain the following for Δr (drop ‘small’ terms)

$$\left\{ \begin{array}{l} \Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2} \Delta\rho \\ \Delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \end{array} \right.$$

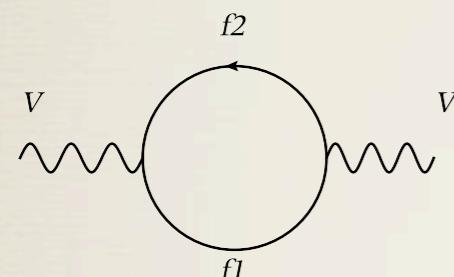
Use optical theorem to relate hadronic vacuum polarization to $e^+e^- \rightarrow \text{hadrons}$

$\Delta\alpha = 0.06649(12)$ (PDG)

$\Delta\varrho$ and non-decoupling

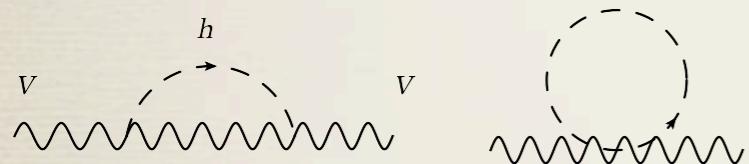
True for any choice of parameters in the Lagrangian:

$$\frac{M_{W0}^2}{M_{Z0}^2 c_{W0}^2} = 1 \Rightarrow \underbrace{\frac{M_W^2}{M_Z^2 c_W^2}}_{\text{finite}} - s_W^2 \underbrace{\Delta\rho}_{\text{finite}} = 1$$



$$\Delta\rho_{ferm} = \underbrace{\frac{3G_F m_t^2}{8\pi^2 \sqrt{2}}}_{\text{quadratic in } m_t} + \text{subleading terms}$$

Exercise: Derive these



$$\Delta\rho_{Higgs} = -\underbrace{\frac{3G_F M_Z^2 s_W^2}{4\pi^2 \sqrt{2}}}_{\text{logarithmic in } M_H} \ln \frac{M_H}{M_Z} + \text{subleading terms}$$

(Veltman screening)

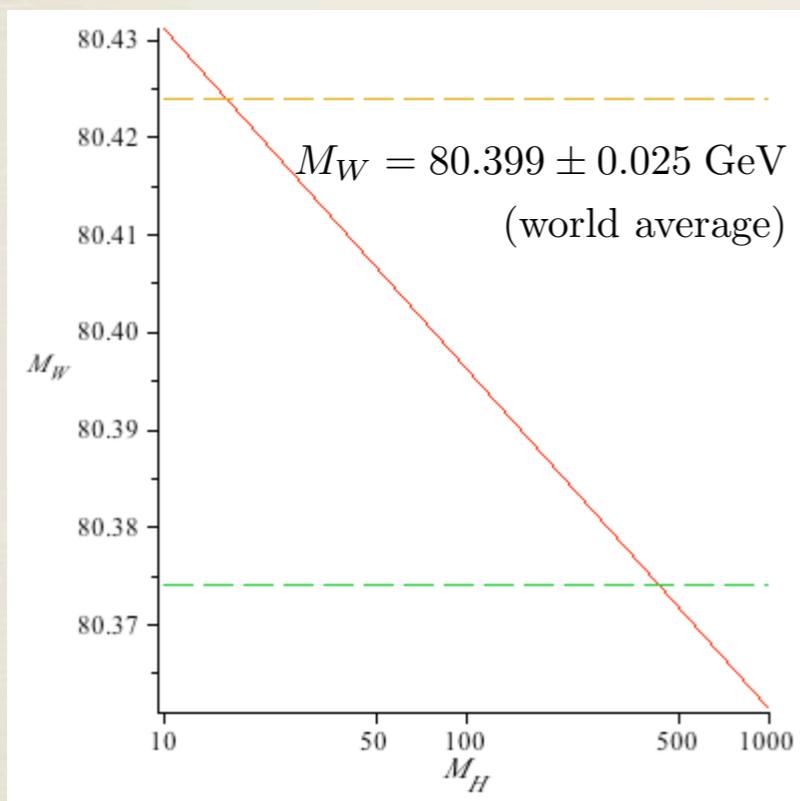
Decoupling theorem holds only if dimensionful parameters made large

$$m_t = \frac{\lambda_t v}{\sqrt{2}} \Rightarrow m_t \rightarrow \infty, \quad v \text{ fixed} \Rightarrow \lambda_t \rightarrow \infty$$

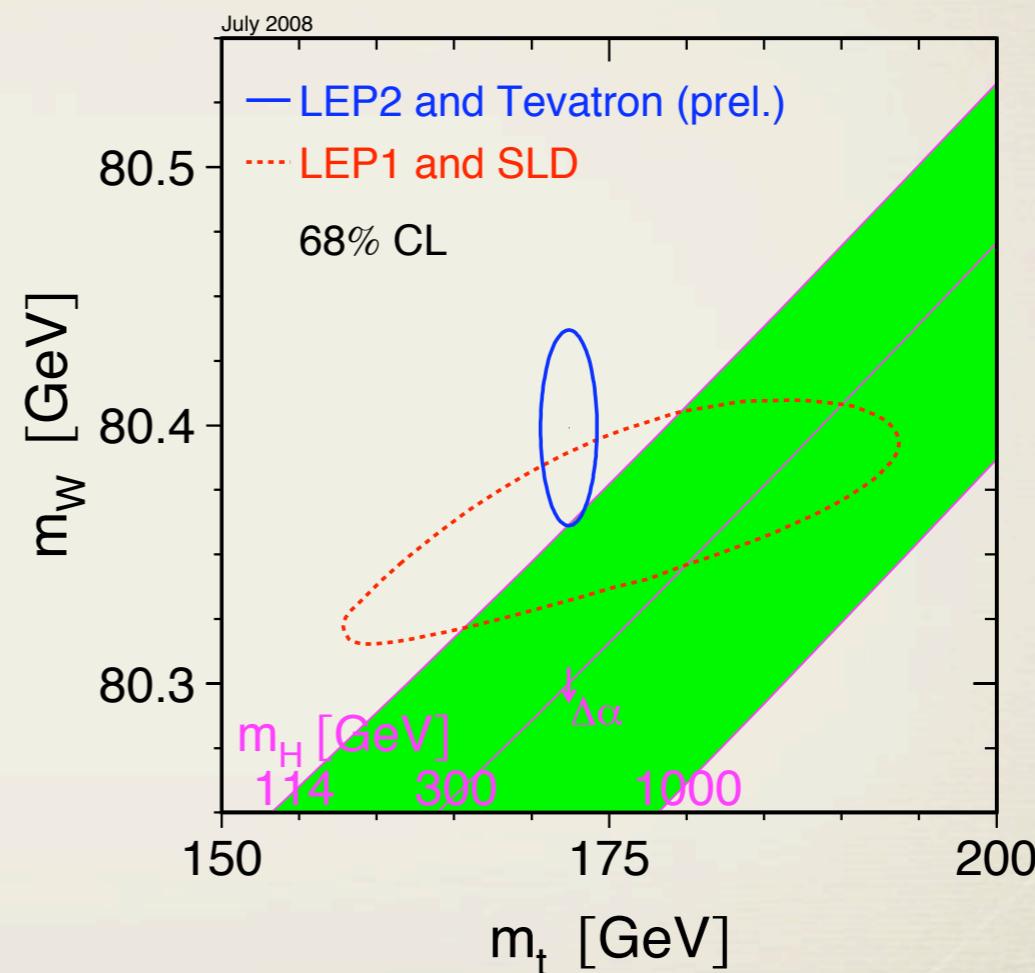
$$M_H^2 = 2\lambda v^2 \Rightarrow M_H \rightarrow \infty, \quad v \text{ fixed} \Rightarrow \lambda \rightarrow \infty$$

Bounding the Higgs mass

- * Logarithmic dependence on M_H allows M_W to bound it (but very sensitive to the top-quark mass)



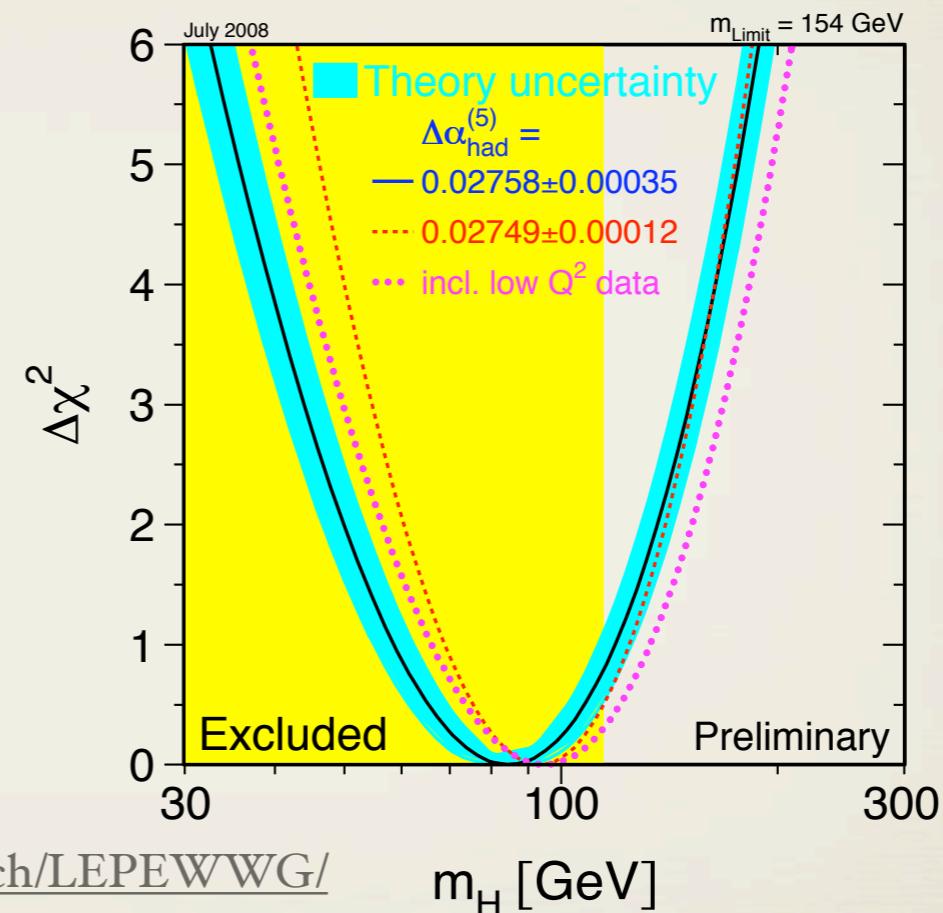
(Refinements needed for real comparison to data; important $\ln(m_t)$ and other terms; see PDG and refs within)



$$M_W^{tree} = 80.94 \text{ GeV} \Rightarrow M_W^{1-loop} = 80.39 \text{ GeV} \quad (M_H = 120 \text{ GeV})$$

Global EW fit

- * Do the same for large set of LEP-SLC measurements



<http://lepewwg.web.cern.ch/LEPEWWG/>

SM Higgs mass: $M_H < \sim 160 \text{ GeV}$ from EW precision measurements

S, T, and hiding a heavy Higgs

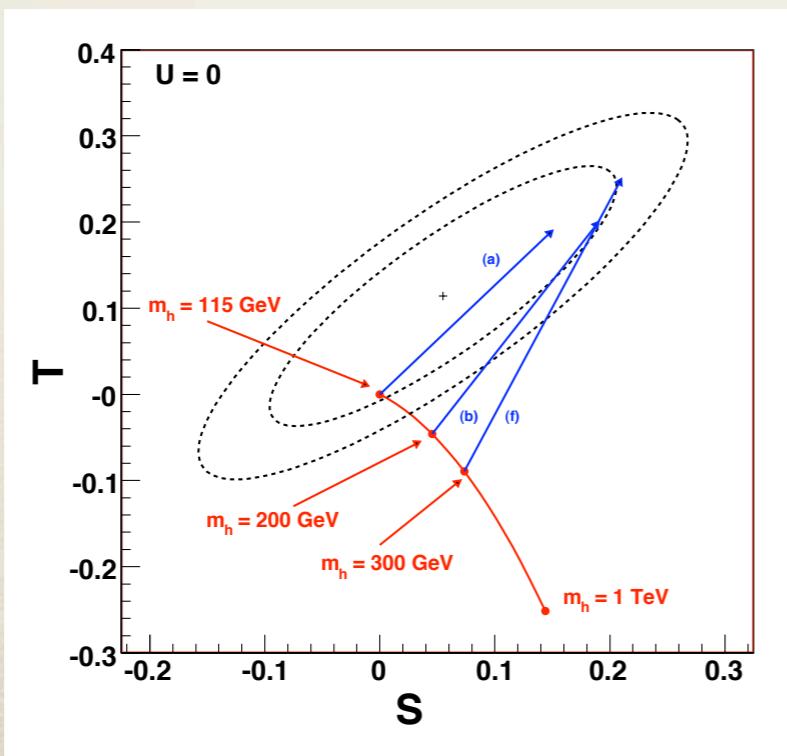
- * How robust are these bounds? Consider corrections that are *oblique*: affect only gauge boson propagators

$$\begin{aligned}\alpha \textcolor{red}{T} &= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \Delta\rho \\ \frac{\alpha}{4s_W^2 c_W^2} \textcolor{red}{S} &= \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}\end{aligned}$$

(Not a complete basis, but these are often the most important ones)

- * Calculate for reference M_H , propagate through all EW parameters

Example: 4th generation
(Kribs et al., 0706.3718)



$$\Delta\rho_{new} = \frac{3G_F \overbrace{\Delta m_{ferm}^2}^{doublet\ mass\ splitting}}{8\pi^2\sqrt{2}} - \frac{3G_F M_Z^2 s_W^2}{4\pi^2\sqrt{2}} \ln \frac{M_H}{M_H^{ref}}$$

⇒ increase M_H , cancel with Δm

Need direct searches!

More in Christophe's lectures

Decays of the Higgs boson

Higgs decays

- * Since $g_{Hxx} \sim m_x$, Higgs tends to decay to heaviest kinematically accessible states (with many important caveats...)
- * Tree-level decays to various massive final states:

$$\Gamma_{qq} = N_c \frac{G_F}{4\sqrt{2}\pi} \textcolor{red}{M_H} m_f^2 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{\textcolor{red}{3/2}}, \quad \Gamma_{ll} = \frac{G_f}{4\sqrt{2}\pi} \textcolor{red}{M_H} m_f^3 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{\textcolor{red}{3/2}}$$
$$\Gamma_{VV} = \frac{G_F}{8\sqrt{2}\pi n_V} \textcolor{red}{M_H^3} (1 - 4x)^{\textcolor{red}{1/2}} (1 - 4x + 12x^3) \text{ with } x = \frac{M_V^2}{M_H^2}, n_W = 1, n_Z = 2$$

- * Threshold structure depends on spin, CP ($\frac{3}{2} \rightarrow \frac{1}{2}$ for CP-odd A)
- * Note $\Gamma_{ff} \sim M_H$, while $\Gamma_{VV} \sim (M_H)^3 \Rightarrow$ when W, Z channels open, Higgs becomes very broad
- * For light Higgs ($M_H \leq 130$ GeV), expect bb, $\tau\tau$, cc to be important

Equivalence theorem

- * Growth of VV width comes from longitudinal gauge modes

$$\mathcal{A}(h \rightarrow W_L^+ W_L^-) = 2 \frac{M_W^2}{v} \epsilon_L^+ \cdot \epsilon_L^-, \quad \epsilon_L^\pm = \frac{E}{M_W} (\pm \beta_W, \vec{0}, 1)$$

$$\mathcal{A}(h \rightarrow W_L^+ W_L^-) \rightarrow -\frac{M_H^2}{v} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right)$$

$$\Gamma_{WW} = \frac{1}{16\pi M_H} |\mathcal{A}|^2 \rightarrow \frac{G_F M_H^3}{8\pi \sqrt{2}} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right)$$

- * In the high energy limit, longitudinal mode interactions equivalent to those of eaten scalar \Rightarrow *Goldstone boson equivalence theorem*

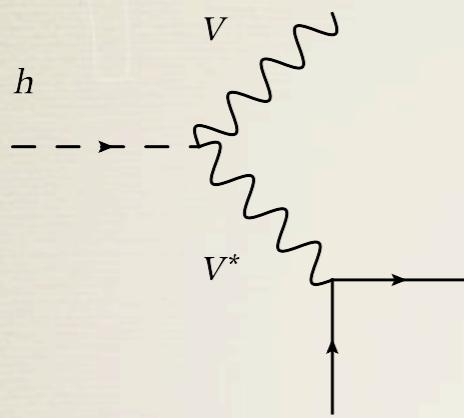
$$= -i \frac{M_H^2}{v}$$

$$\mathcal{A}(h \rightarrow \phi^+ \phi^-) = -\frac{M_H^2}{v}$$

Exercise: Work out from L_{Higgs}

Three-body decays

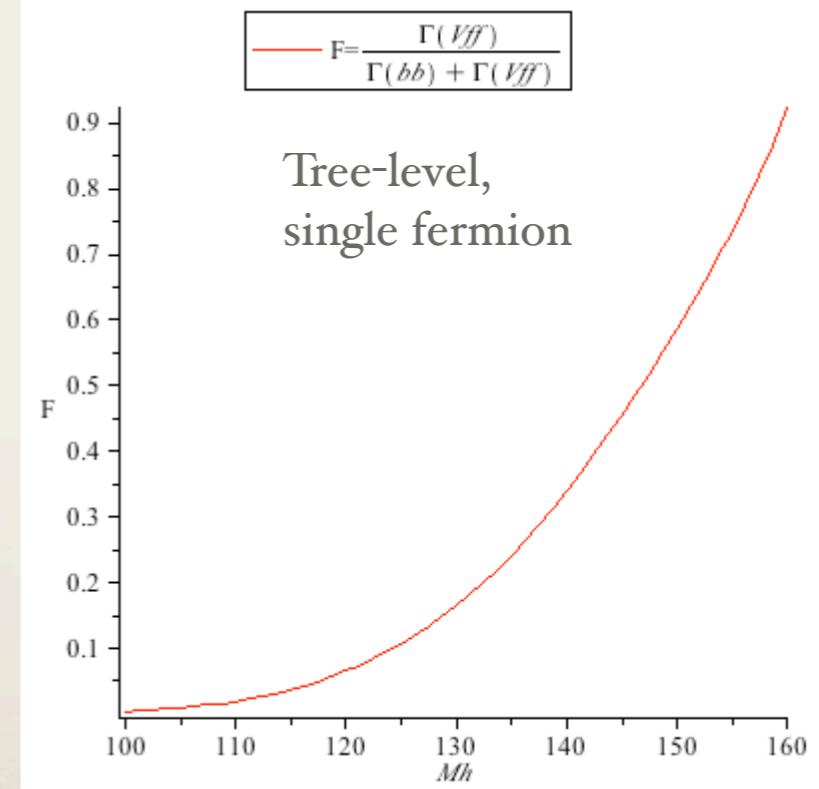
- * Since $M_{W,Z} \gg m_{b,c,\tau}$, $H \rightarrow VV^* \rightarrow V\bar{f}f$ important for $M_H < 2M_{W,Z}$



$$\Gamma_{W\bar{f}f} = \frac{3G_F^2 M_W^4}{16\pi^3} M_H \left\{ \frac{3(1 - 8x + 20x^2)}{\sqrt{4x - 1}} \arccos \left(\frac{3x - 1}{2x^{3/2}} \right) - \frac{1 - x}{2x} (2 - 13x + 47x^2) - \frac{3}{2} (1 - 6x + 4x^2) \ln x \right\}$$

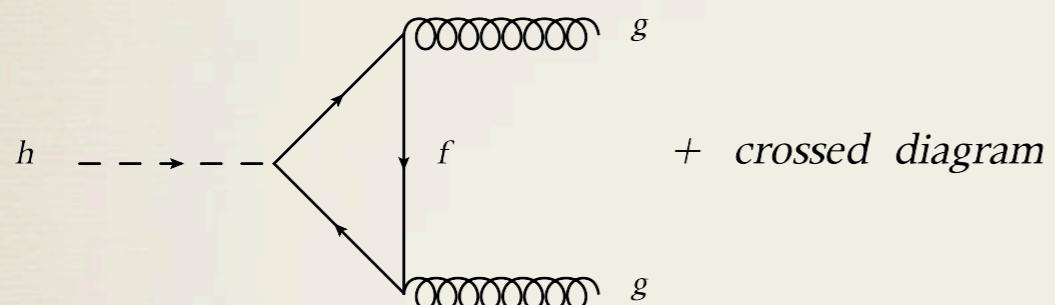
$$x = M_W^2 / M_H^2$$

- * Important mode even down at $M_H \approx 130$ GeV since $f = e, \mu$



Loop-induced $H \rightarrow gg$

- * Can we leverage the large Htt , HVV couplings at low M_H ?
- * Two important cases: $h \rightarrow gg$ (production more important), $h \rightarrow \gamma\gamma$



$$\Gamma_{gg} = \frac{G_F \alpha_s^2 M_H^3}{36\pi^3 \sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \quad \text{with } \tau_Q = \frac{M_H^2}{4m_Q^2}$$

$$\begin{aligned} \mathcal{F}_{1/2}(\tau) &= \frac{2}{\tau^2} [\tau + (\tau - 1)f(\tau)] \\ f(\tau) &= \begin{cases} \arcsin^2 \sqrt{\tau} & \tau \leq 1 \\ -\frac{1}{4} \left[\ln \frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi \right]^2 & \tau > 1 \end{cases} \end{aligned}$$

$$\tau \rightarrow 0 \Rightarrow \mathcal{F}_{1/2} \rightarrow \frac{4}{3}$$

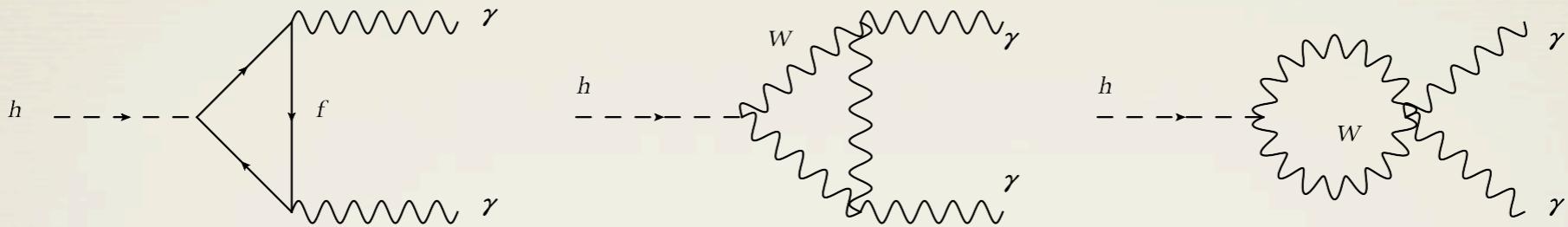
$$\tau \rightarrow \infty \Rightarrow \mathcal{F}_{1/2} \rightarrow -\frac{2m_Q^2}{M_H^2} \ln \frac{M_H^2}{m_Q^2}$$

- Independent of m_f when $m_f \rightarrow \infty \Rightarrow$ true for *any* heavy fermion that gets its mass from Higgs
- Although $m_b < m_t$, can give non-negligible destructive interference due to log (~5%)

Exercise: Derive $m_t \rightarrow \infty$ result from direct integration

Loop-induced $H \rightarrow \gamma\gamma$

- * Crucial for low-mass Higgs search at LHC

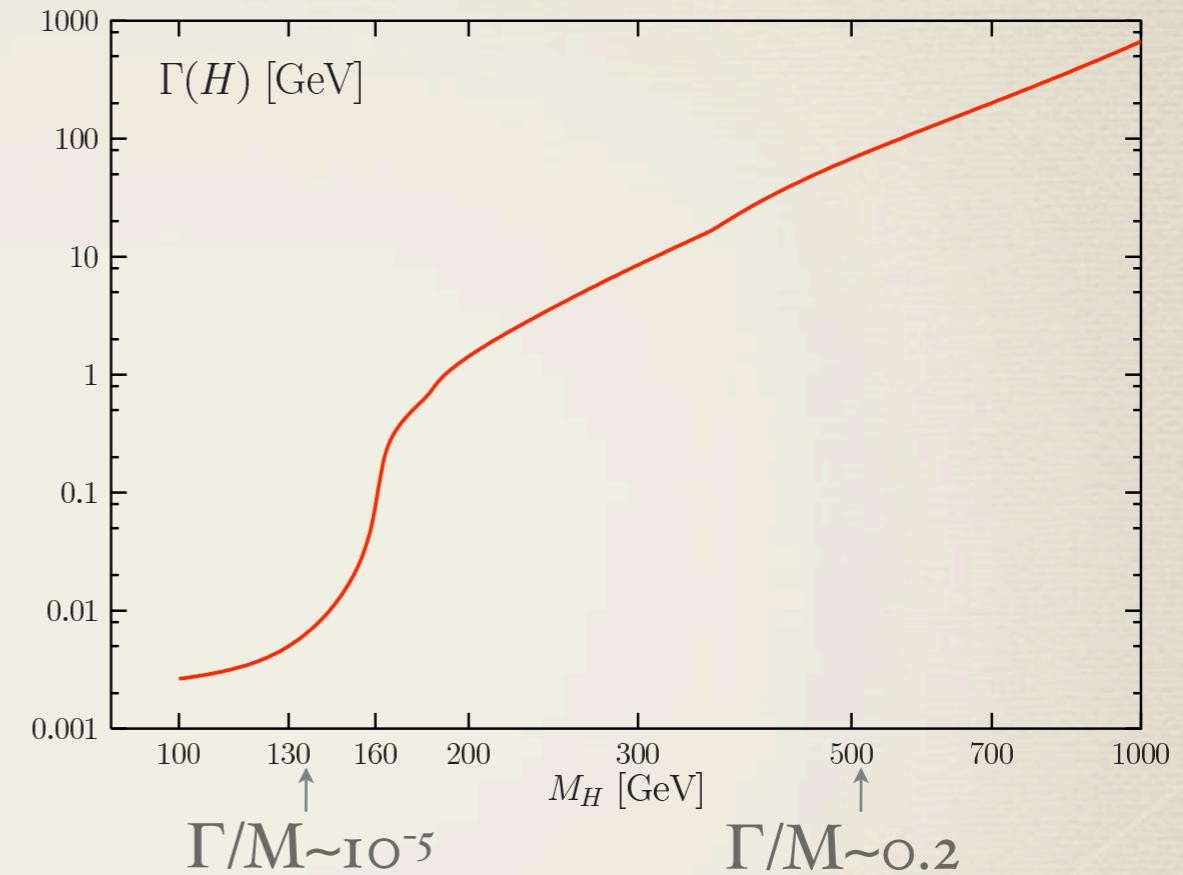
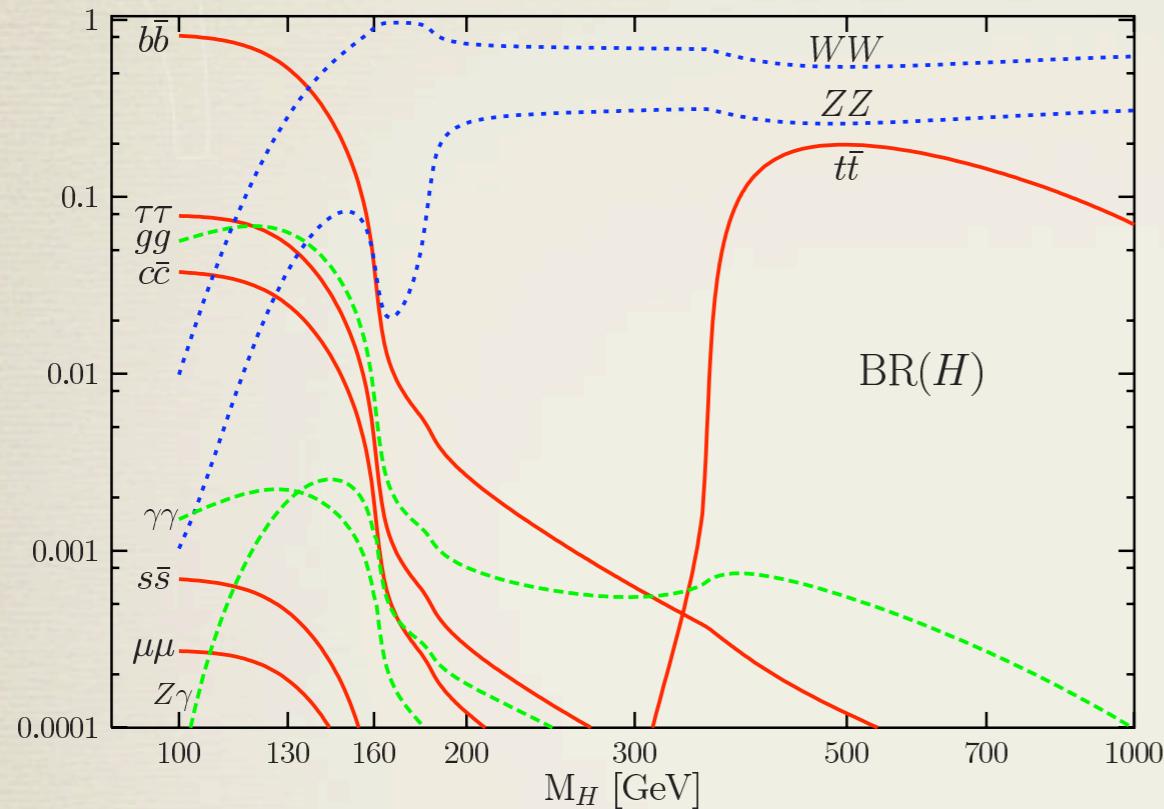


$$\Gamma_{\gamma\gamma} = \frac{G_F \alpha^2 M_H^3}{128\pi^3 \sqrt{2}} \left| \sum_f N_c Q_f^2 \mathcal{F}_{1/2}(\tau_f) + \mathcal{F}_1(\tau_W) \right|^2$$
$$\mathcal{F}_1(\tau) = -\frac{1}{\tau^2} [2\tau^2 + 3\tau + 3(2\tau - 1)f(\tau)]$$

$$\tau \rightarrow 0 \Rightarrow \mathcal{F}_1 \rightarrow -7$$

W contribution larger than top-quark, they interfere destructively

Putting it all together



Most important channels:

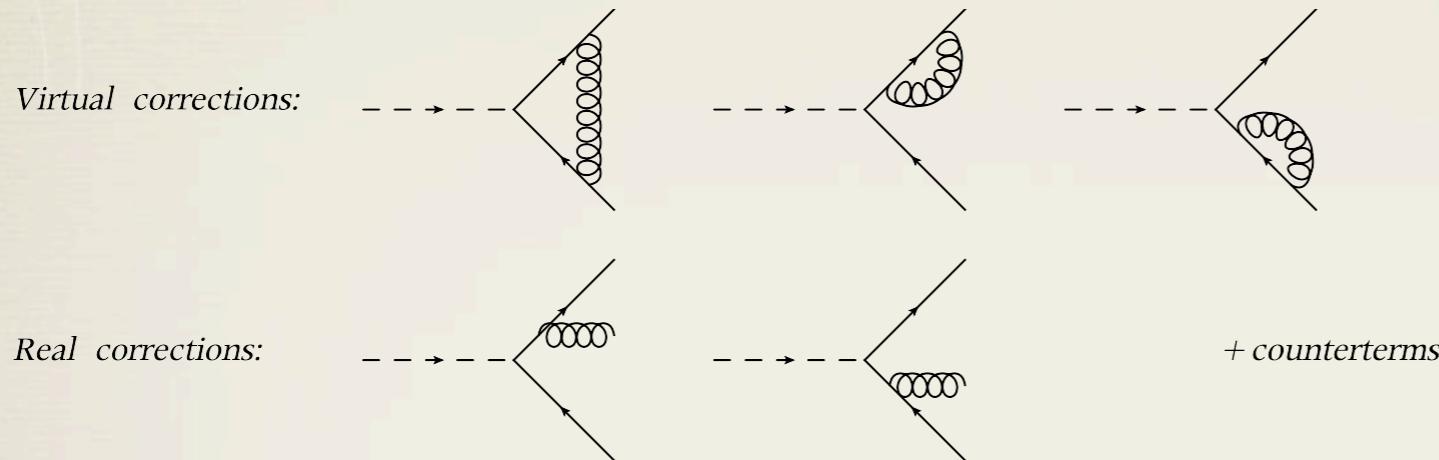
$M_H \leq 130$ GeV: bb , $\tau\tau$, $\gamma\gamma$ (clean signature)

$M_H \geq 130$ GeV: WW , ZZ

(boundaries are rough)

Refinement: heavy-quark decays

- * Which mass to use in $\Gamma_{bb,cc}$; pole mass, MS-bar?



- * Pole scheme calculation (on-shell counterterm used):

$$\Gamma_{qq}^{NLO} = N_c \frac{G_F}{4\sqrt{2}\pi} M_H m_q^2 \left[1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left(\frac{9}{4} + \underbrace{\frac{3}{2} \ln \frac{m_q^2}{M_H^2}}_{large} \right) \right] + \mathcal{O}(m_q^2/M_H^2) \quad (\text{pole scheme})$$

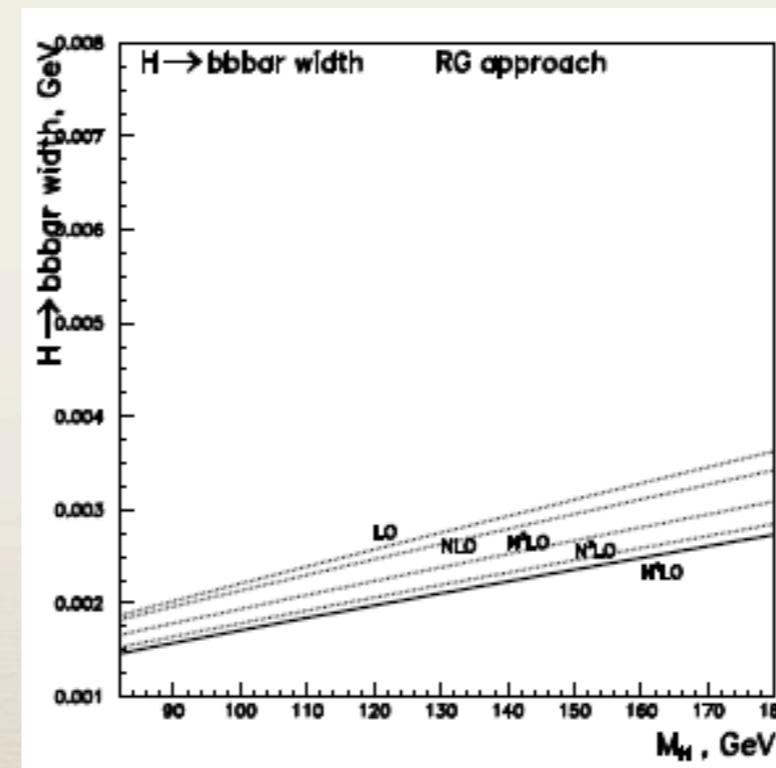
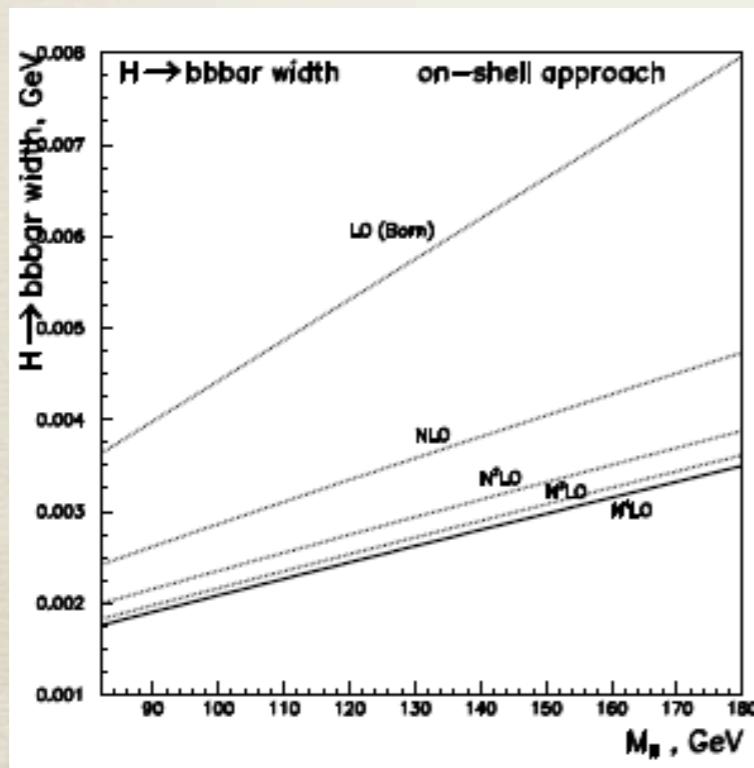
Negative for $m_q \sim 10$ MeV

- * Log comes only from counterterms (KLN theorem applied to $\text{Im}\{\Pi(M_H)\}$ requires this)

Translation to running mass

- * Translate from pole \rightarrow MSbar scheme (leading terms only)

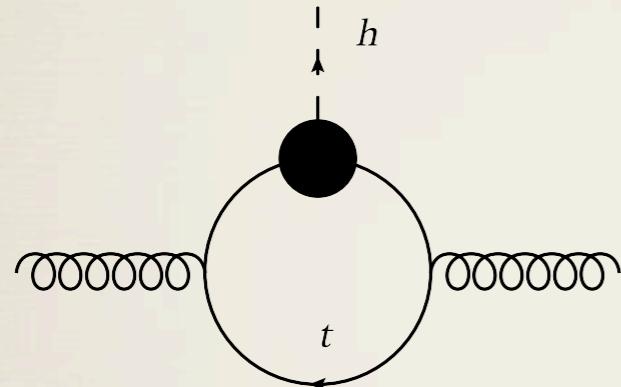
$$\begin{aligned}
 m_q &= \bar{m}_q(\bar{m}_q) \left\{ 1 + \frac{4}{3} \frac{\alpha_s}{\pi} \right\} \quad (\text{derive this}) \\
 \bar{m}_q(\bar{m}_q) &= \bar{m}_q(M_H) \left\{ 1 - \frac{\alpha_s}{\pi} \ln \frac{M_H^2}{m_q^2} \right\} \quad (\text{standard RGE}) \\
 \Rightarrow \Gamma_{qq}^{NLO} &= N_c \frac{G_F}{4\sqrt{2}\pi} M_H \bar{m}_q^2(M_H) \left[1 + \frac{17}{3} \frac{\alpha_s}{\pi} \right] \quad (\text{MSbar scheme})
 \end{aligned}$$



(from Kataev, Kim 0902.1442,
can get other literature there)

Refinement: low-energy theorems

- * Can exactly calculate QCD corrections to $h \rightarrow gg, \gamma\gamma$ (two-loop diagrams plus real radiation for gg decay) Djouadi, Spira, Zerwas early 1990s
- * Useful, illuminating alternative approach for $2m_t > M_H$



$$\begin{aligned} \frac{i}{k - m_t} &\rightarrow \frac{i}{k - m_t} \frac{-im_t}{v} \frac{i}{k - m_t} = i \frac{m_t}{v} \left(\frac{1}{k - m_t} \right)^2 \\ &= \frac{m_t}{v} \frac{\partial}{\partial m_t} \frac{i}{k - m_t} \end{aligned}$$

Generates both diagrams in the $M_H \rightarrow 0$ limit

- * Diagrammatically, clear that Higgs interaction comes from derivatives of the top part of the gluon self-energy:

$$\mathcal{M}(hgg) \underset{p_H \rightarrow 0}{=} \frac{m_t}{v} \frac{\partial}{\partial m_t} \mathcal{M}(gg)$$

Effective Lagrangian

- * Integrate out top quark to produce effective Lagrangian

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} + \mathcal{L}_{top} \\ \Rightarrow \underbrace{G_{\mu}^{a\prime}}_{\text{EFT field}} &= \zeta_3 \underbrace{G_{\mu}^a}_{\text{full QCD}} \\ \Rightarrow \mathcal{L}_{EFT} &= -\frac{\zeta_3}{4} G_{\mu\nu}^{a\prime} G_a^{\mu\nu} \end{aligned}$$

Equate propagators in full QCD and EFT

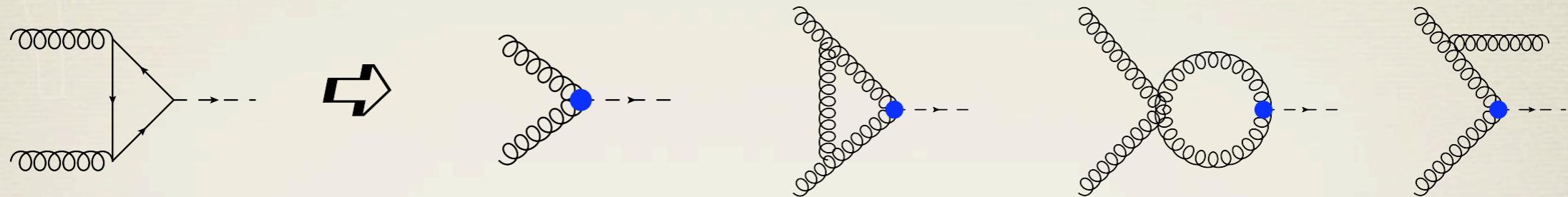
$$\begin{aligned} -\frac{ig_{\mu\nu}}{p^2} \zeta_3 &= -\frac{ig_{\mu\nu}}{p^2} \underbrace{[1 + \Pi_t(0)]}_{m_t^2 \gg p^2} \\ \Rightarrow \zeta_3 &= 1 + \Pi_t(0) \\ \Rightarrow \mathcal{L}_{EFT} &= -\frac{[1 + \Pi_t(0)]}{4} G_{\mu\nu}^{a\prime} G_a^{\mu\nu} \end{aligned}$$

- * Can generate hgg amplitudes from derivatives of gg amplitudes:

$$\begin{aligned} \mathcal{L}_{EFT}^{hgg} &= -\frac{m_t}{4v} \left(\frac{\partial}{\partial m_t} \Pi_t(0) \right) h G_{\mu\nu}^{a\prime} G_a^{\mu\nu} \\ \Rightarrow \Pi_t(0) &= \frac{\alpha_s}{6\pi} \left[\frac{\bar{\mu}^2}{m_t^2} \right]^\epsilon \frac{\Gamma(1+\epsilon)}{\epsilon} \\ \Rightarrow \boxed{\mathcal{L}_{EFT}^{hgg} = \frac{\alpha_s}{12\pi v} h G_{\mu\nu}^{a\prime} G_a^{\mu\nu}} & \quad | \quad \boxed{|} \end{aligned}$$

Decay in the EFT

- * Reduces 2-loop calculation \Rightarrow 1-loop; separates m_t dependence



- * Systematically improvable to all orders in α_s

$$\begin{aligned}\mathcal{L}_{EFT}^{hgg} &= -C_1 \frac{h}{v} G_{\mu\nu}^{a'} G_a^{\mu\nu'} \\ C_1 &= -\frac{1}{12} \frac{\alpha_s}{\pi} \left\{ 1 + \frac{\alpha_s}{\pi} \left(\frac{11}{4} - \frac{1}{6} \ln \frac{\mu^2}{m_t^2} \right) + \dots \right\}\end{aligned}$$

Correction to $h \rightarrow$ light hadrons:
(must include qq at higher orders)

$$\begin{aligned}K &= 1 + \underline{17.9167} a'_s + 152.5(a'_s)^2 + 381.5(a'_s)^3 \\ &= 1 + \underline{0.65038} + 0.20095 + 0.01825.\end{aligned}$$

Large!

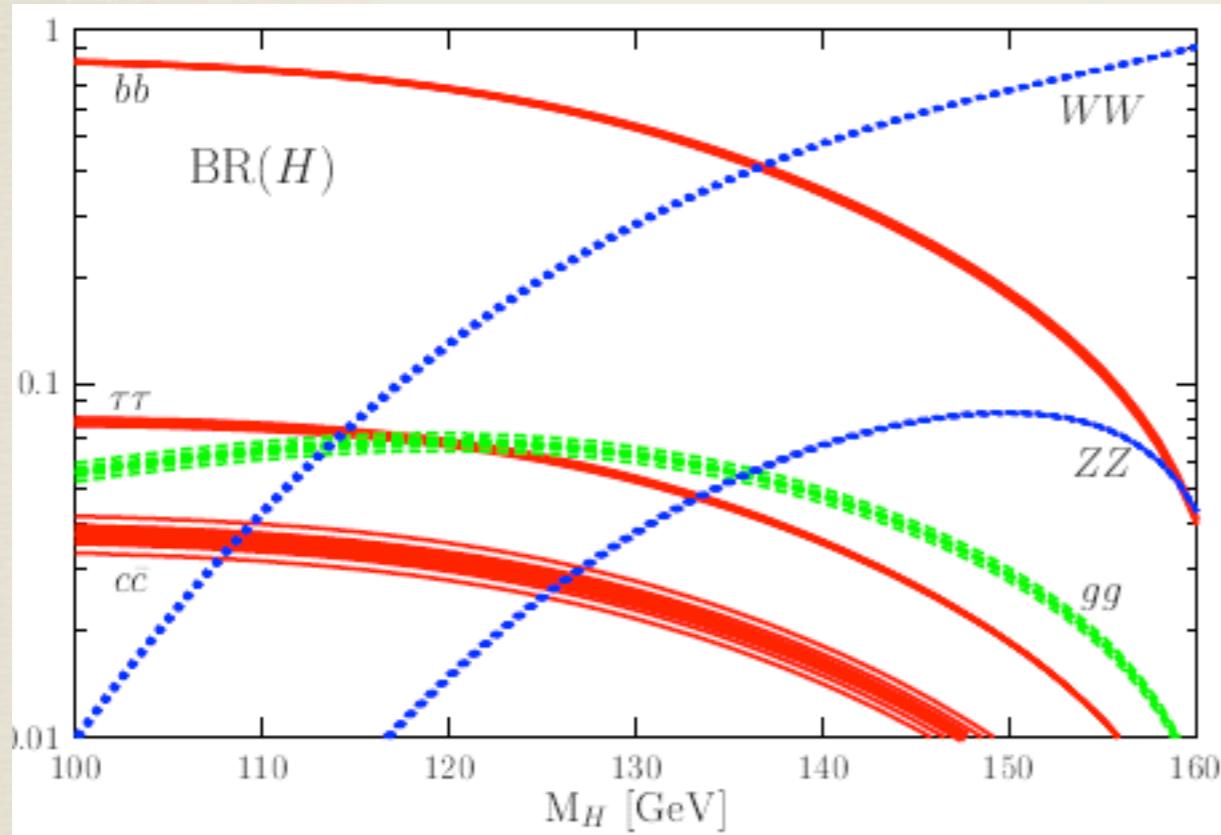
Baikov, Chetyrkin
hep-ph/0604194

- * Can do same for $h \rightarrow \gamma\gamma$ decay, for W contribution also

For references and subtleties, see Chetyrkin et al. hep-ph/9708255, Kniehl, Spira hep-ph/9504378

Higgs decays with errors

- * Important when considering measurement of Higgs properties



from Djouadi hep-ph/0503172

Major uncertainties: $\alpha_s(\mu)$, $\bar{m}_c(\mu)$

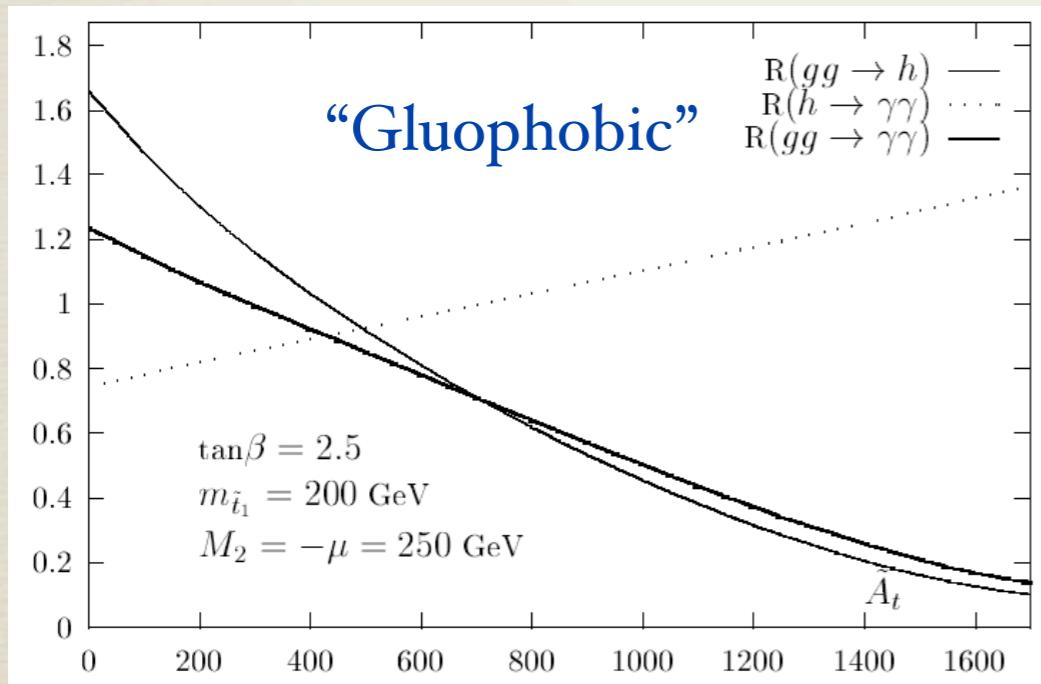
To my knowledge, not updated to include improved extraction of m_c

Kuhn, Steinhauser, Sturm hep-ph/0702103

- * HDECAY: SM and SUSY decays Djouadi, Kalinowski, Spira

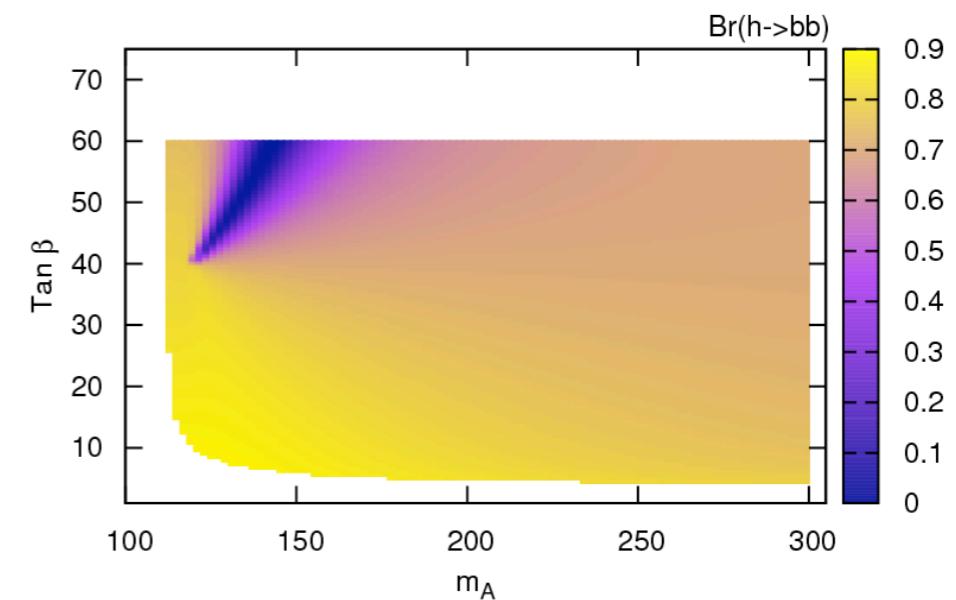
Decays beyond the SM

- * MSSM: cancellations between top/squark loops in gg , $\gamma\gamma$ modes
- * MSSM: opposite signs for A_t , μ terms \Rightarrow suppresses $h \rightarrow bb$



Djouadi, hep-ph/9806315

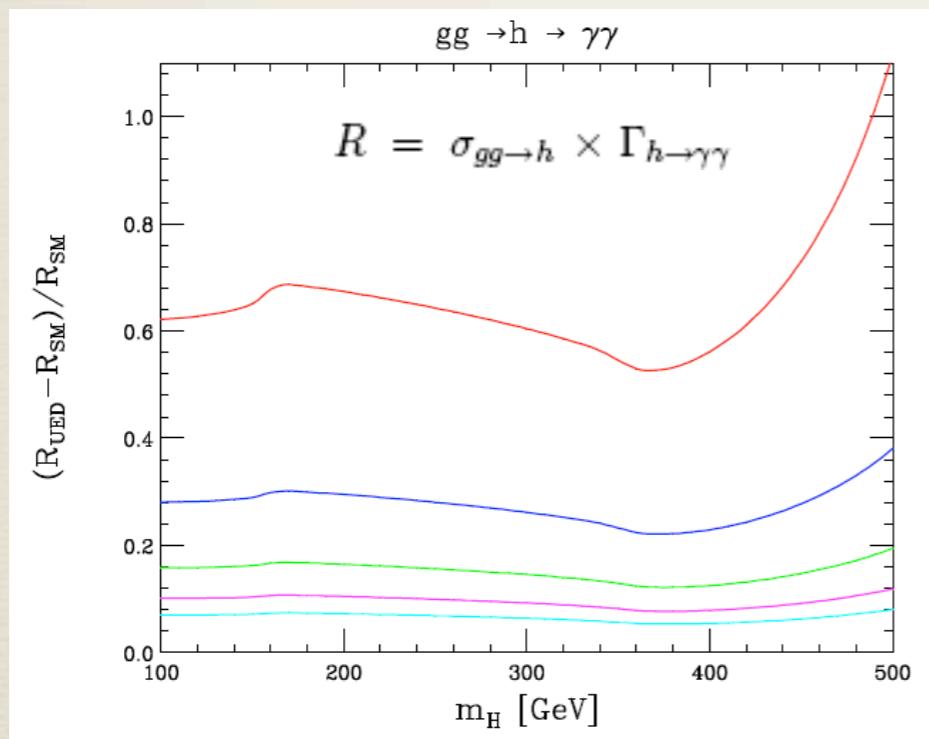
$$\left[\frac{m_A^2}{m_Z^2} - \frac{1}{2\pi^2} (3\bar{\mu}^2 - \bar{\mu}^2 \bar{A}_t^2) + 1 \right] \simeq \frac{\tan\beta}{150} \left[\bar{\mu} \bar{A}_t (2\bar{A}_t^2 - 11) \right]$$



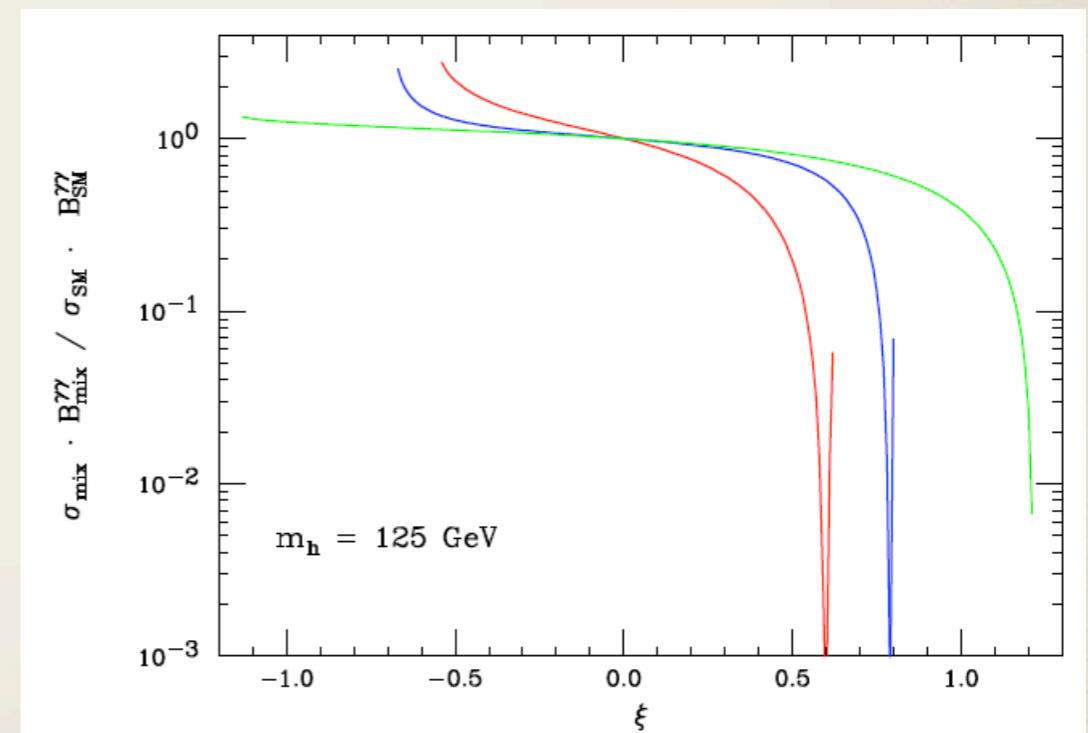
Draper, Liu, Wagner 0905.4721

Decays beyond the SM

- * Flat ED: effects from Kaluza-Klein modes propagating in loop
- * Warped ED: mixing with new scalar radion fields that stabilizes 5-th dimension



FP, hep-ph/0204067



Hewett, Rizzo hep-ph/0202155

Decays beyond the SM

- * NMSSM: decays to light CP-odd scalar can produce final states
 $h \rightarrow aa \rightarrow bb\tau\tau, \tau\tau\tau\tau, \tau\tau\gamma\gamma, \dots$
- * Extended scalar sectors: decays to stable scalars (dark matter) can make Higgs invisible decaying

m_{h_1}/m_{a_1} (GeV)	Branching Ratios			$n_{\text{obs}}/n_{\text{exp}}$ units of 1σ	$s95$	N_{SD}^{LHC}
	$h_1 \rightarrow b\bar{b}$	$h_1 \rightarrow a_1 a_1$	$a_1 \rightarrow \tau\bar{\tau}$			
98.0/2.6	0.062	0.926	0.000	2.25/1.72	2.79	1.2
100.0/9.3	0.075	0.910	0.852	1.98/1.88	2.40	1.5
100.2/3.1	0.141	0.832	0.000	2.26/2.78	1.31	2.5
102.0/7.3	0.095	0.887	0.923	1.44/2.08	1.58	1.6
102.2/3.6	0.177	0.789	0.814	1.80/3.12	1.03	3.3
102.4/9.0	0.173	0.793	0.875	1.79/3.03	1.07	3.6
102.5/5.4	0.128	0.848	0.938	1.64/2.46	1.24	2.4
105.0/5.3	0.062	0.926	0.938	1.11/1.52	2.74	1.2

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}m_S^2 S^2 - \frac{k}{2}|H|^2 S^2 - \frac{h}{4!}S^4$$

$h \rightarrow SS$ decays can dominate

Burgess, Pospelov, ter Veldhuis NPB 619 (2001); Davoudiasl et al. hep-ph/0405097

Dermisek, Gunion hep-ph/0510322

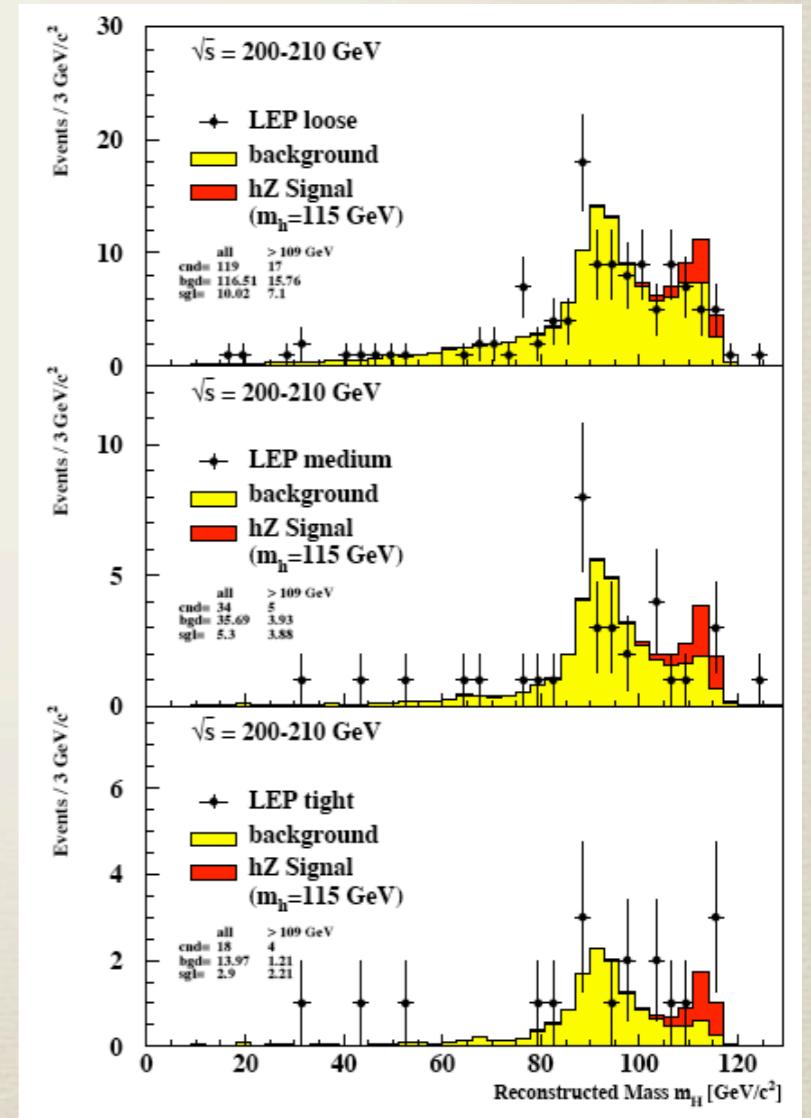
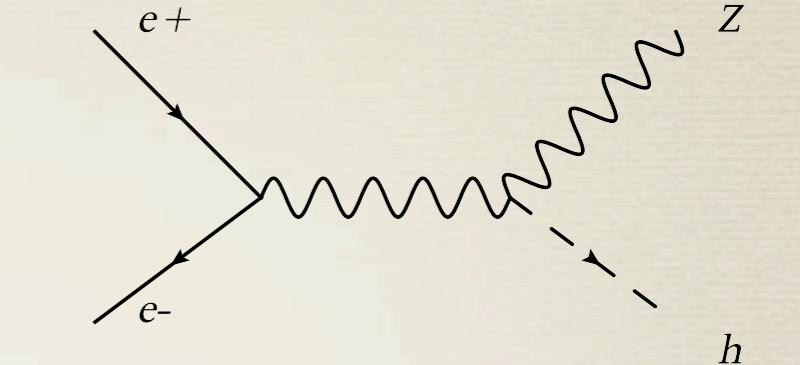
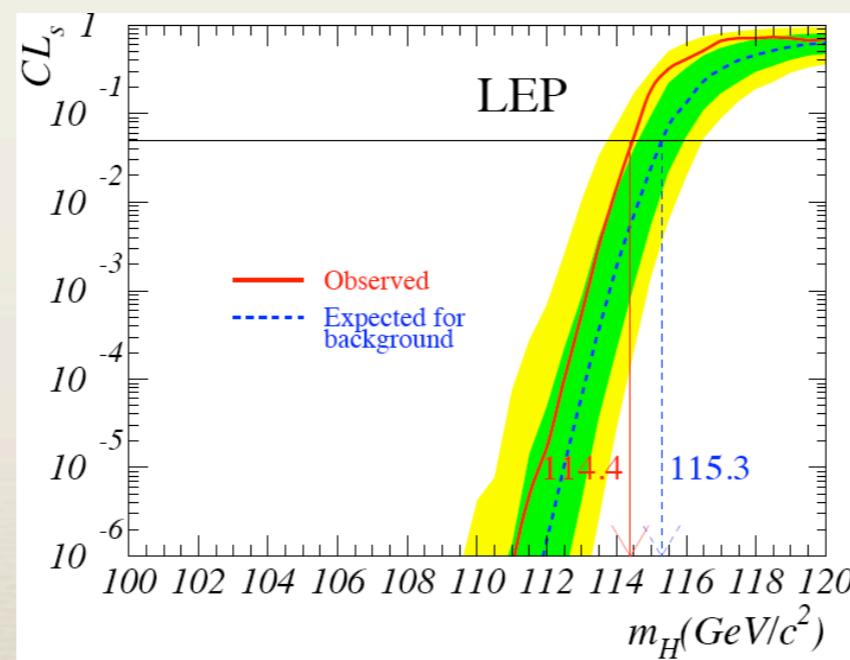
Many deviations, some drastic, from SM predictions possible!

Producing the Higgs boson

Direct searches at LEP

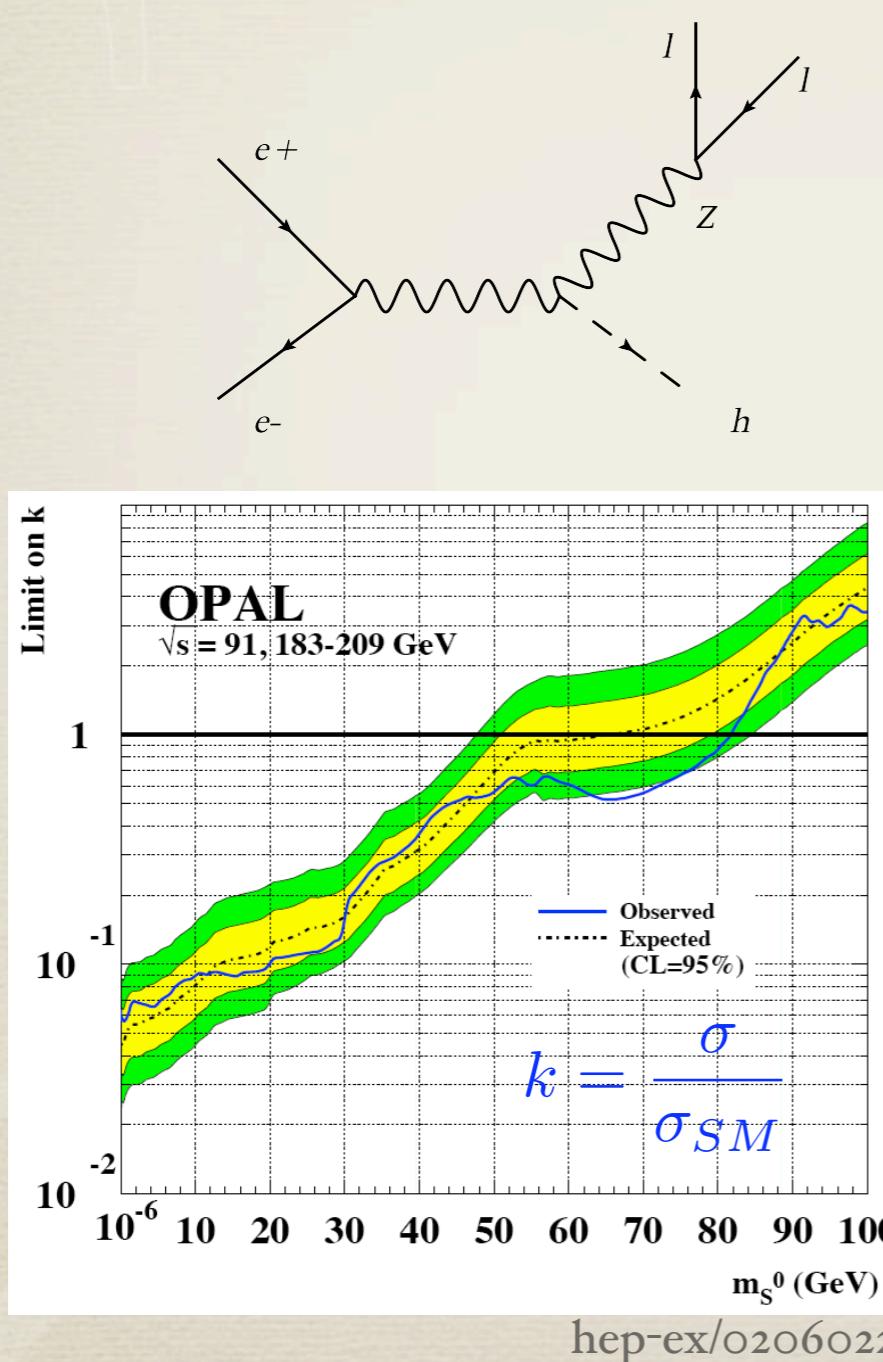
- * LEP2 ran at energies reaching $\sqrt{s} \leq 209$ GeV
- * Dominant production process: $e^+e^- \rightarrow hZ$
- * SM analysis utilizes the following channels:
 - $h \rightarrow bb, Z \rightarrow qq$
 - $h \rightarrow bb, Z \rightarrow \nu\nu$
 - $h \rightarrow bb, Z \rightarrow ll$ ($l=e,\mu$)
 - $h \rightarrow bb, Z \rightarrow \tau\tau$
 - $h \rightarrow \tau\tau, Z \rightarrow qq$

M_H>114.4 GeV



Model-independent search

- * This is optimized for SM decays, any way to remove this bias?



Measure two leptons in final state,
demand they reconstruct to Z mass

$$\begin{aligned} p_{e^+} + p_{e^-} &= p_{l^+} + p_{l^-} + p_X \\ &= p_{ll}^{rec} + p_X \\ \Rightarrow M_X^2 &= s - 2E_{ll} + M_{ll}^2 \end{aligned}$$

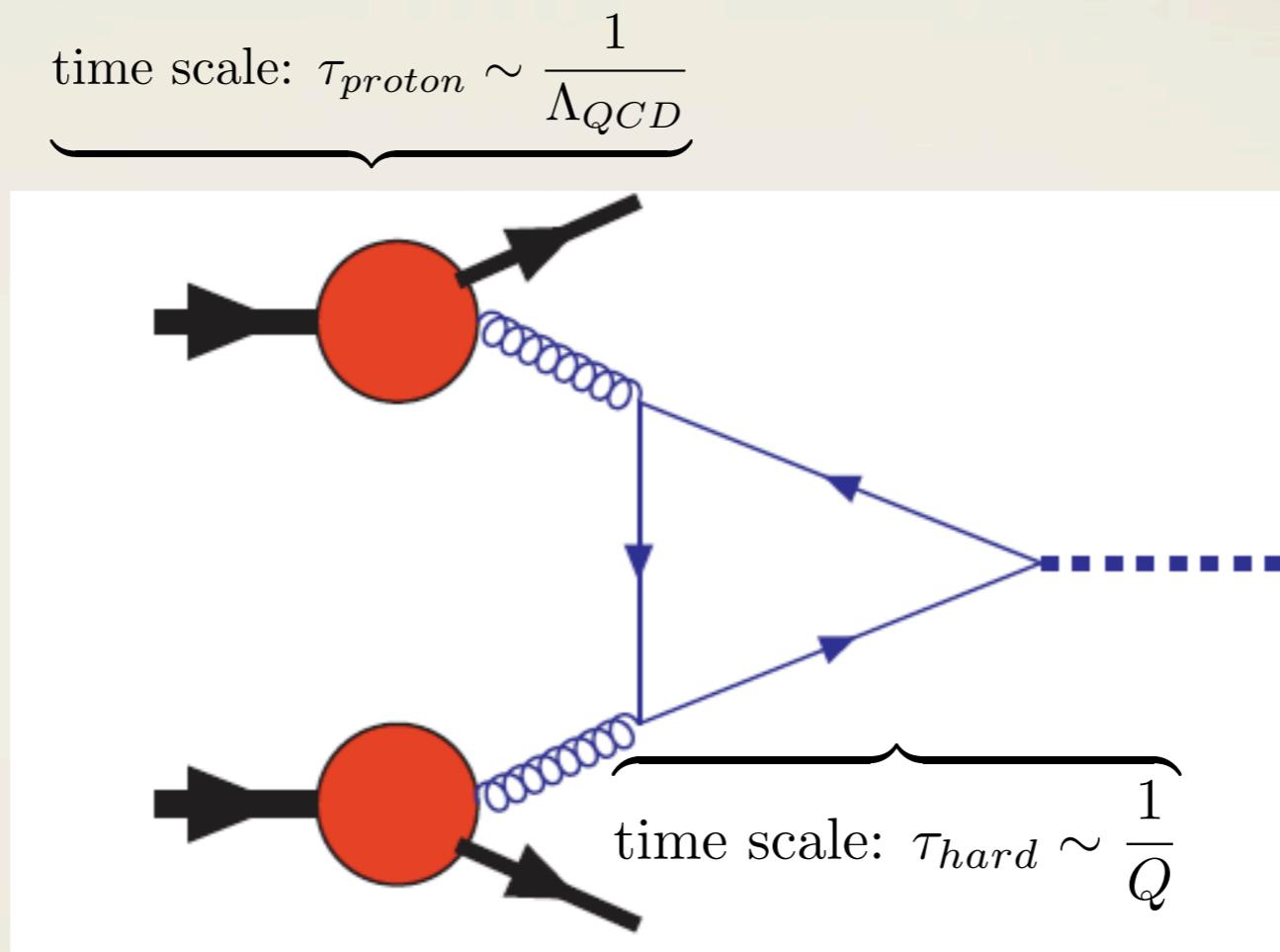
Predicted peak : $M_X^2 = M_H^2$

Limits hold for *any* decay mode

Many other searches
designed for specific models

Hadron collider basics

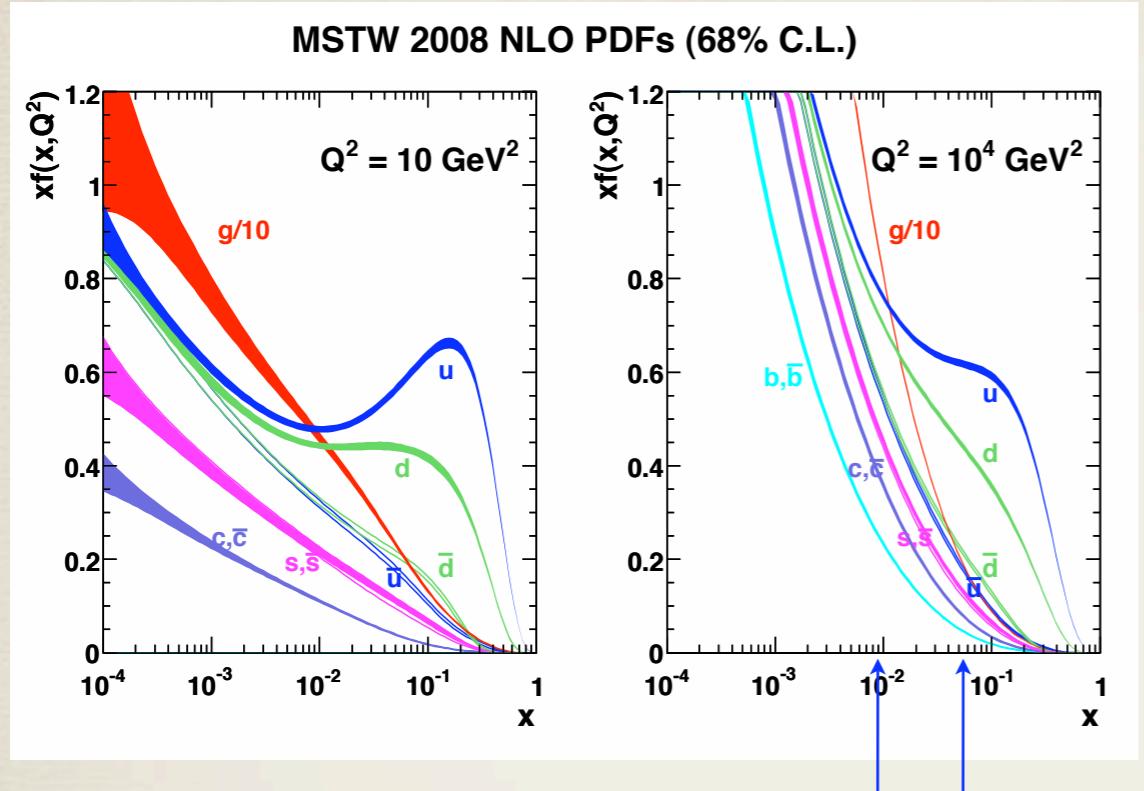
- * The basic picture of hadronic collisions: factorize long and short time processes



$$\sigma_{h_1 h_2 \rightarrow X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \mu_F^2)}_{PDFs} \underbrace{f_{h_2/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \rightarrow X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n$$

power corrections

Parton distribution functions



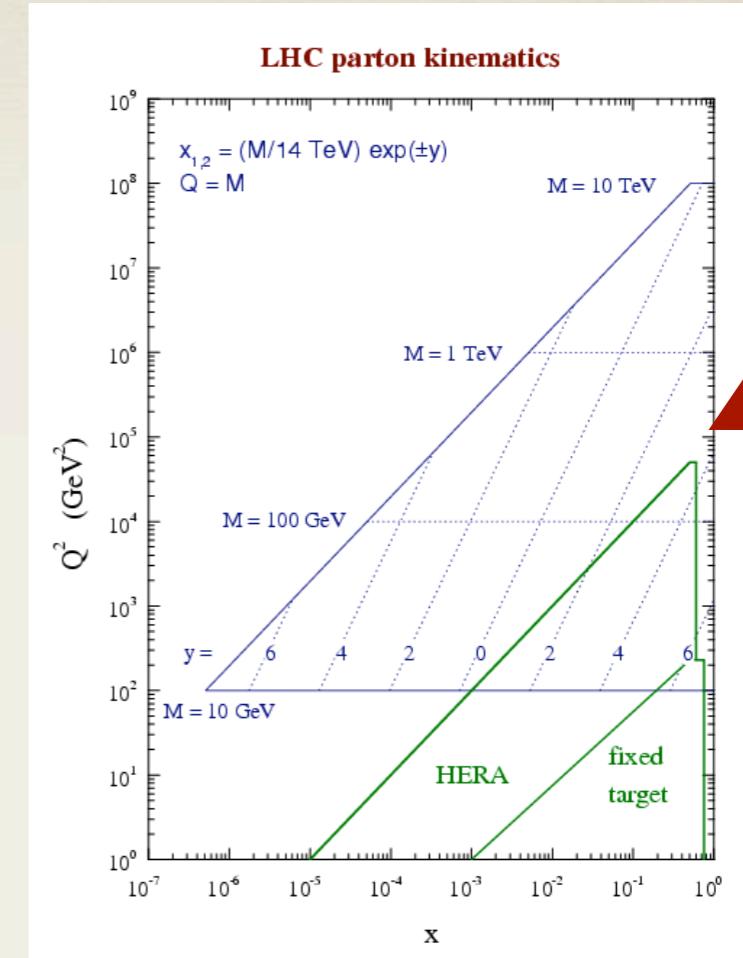
LHC Tevatron

$$x \sim M_H / \sqrt{s}$$

Lots of gluons, at LHC especially

MSTW 2008

Only known at NLO →

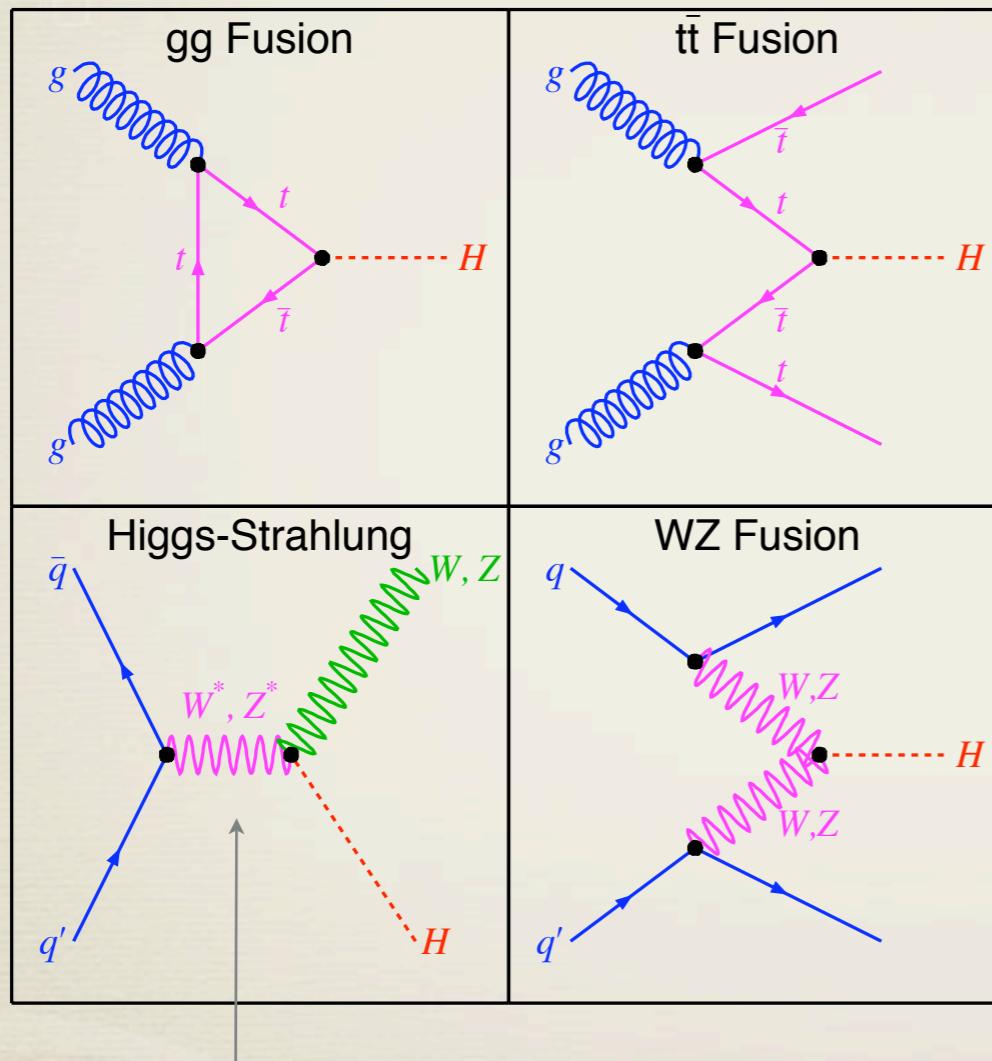


Tev HERA Fixed target

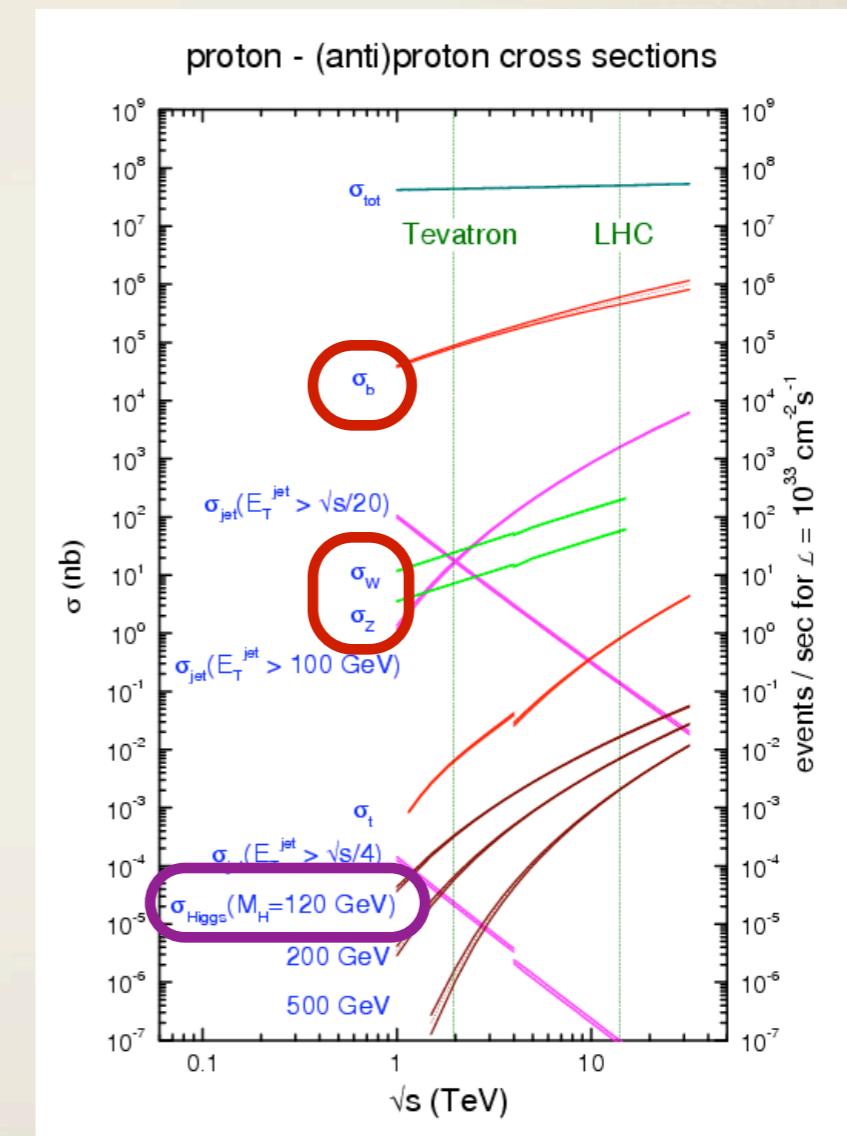
Process	Subprocess	Partons	x range
$\ell^\pm \{p, n\} \rightarrow \ell^\pm X$	$\gamma^* q \rightarrow q$	q, \bar{q}, g	$x \gtrsim 0.01$
$\ell^\pm n/p \rightarrow \ell^\pm X$	$\gamma^* d/u \rightarrow d/u$	d/u	$x \gtrsim 0.01$
$pp \rightarrow \mu^+ \mu^- X$	$u\bar{u}, d\bar{d} \rightarrow \gamma^*$	\bar{q}	$0.015 \lesssim x \lesssim 0.35$
$p\bar{n}/p\bar{p} \rightarrow \mu^+ \mu^- X$	$(u\bar{d})/(u\bar{u}) \rightarrow \gamma^*$	\bar{d}/\bar{u}	$0.015 \lesssim x \lesssim 0.35$
$\nu(\bar{\nu}) N \rightarrow \mu^-(\mu^+) X$	$W^* q \rightarrow q'$	q, \bar{q}	$0.01 \lesssim x \lesssim 0.5$
$\nu N \rightarrow \mu^- \mu^+ X$	$W^* s \rightarrow c$	s	$0.01 \lesssim x \lesssim 0.2$
$\bar{\nu} N \rightarrow \mu^+ \mu^- X$	$W^* \bar{s} \rightarrow \bar{c}$	\bar{s}	$0.01 \lesssim x \lesssim 0.2$
$e^\pm p \rightarrow e^\pm X$	$\gamma^* q \rightarrow q$	g, q, \bar{q}	$0.0001 \lesssim x \lesssim 0.1$
$e^+ p \rightarrow \bar{\nu} X$	$W^+ \{d, s\} \rightarrow \{u, c\}$	d, s	$x \gtrsim 0.01$
$e^\pm p \rightarrow e^\pm c\bar{c} X$	$\gamma^* e \rightarrow e, \gamma^* g \rightarrow c\bar{c}$	c, g	$0.0001 \lesssim x \lesssim 0.01$
$e^\pm p \rightarrow \text{jet} + X$	$\gamma^* g \rightarrow q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
$p\bar{p} \rightarrow \text{jet} + X$	$gg, qg, qq \rightarrow 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
$p\bar{p} \rightarrow (W^\pm \rightarrow \ell^\pm \nu) X$	$ud \rightarrow W, \bar{u}\bar{d} \rightarrow W$	u, d, \bar{u}, \bar{d}	$x \gtrsim 0.05$
$p\bar{p} \rightarrow (Z \rightarrow \ell^+ \ell^-) X$	$uu, dd \rightarrow Z$	d	$x \gtrsim 0.05$

Higgs at hadron colliders

- * Clearly want to use large gluon luminosity; W, Z assisted production another option



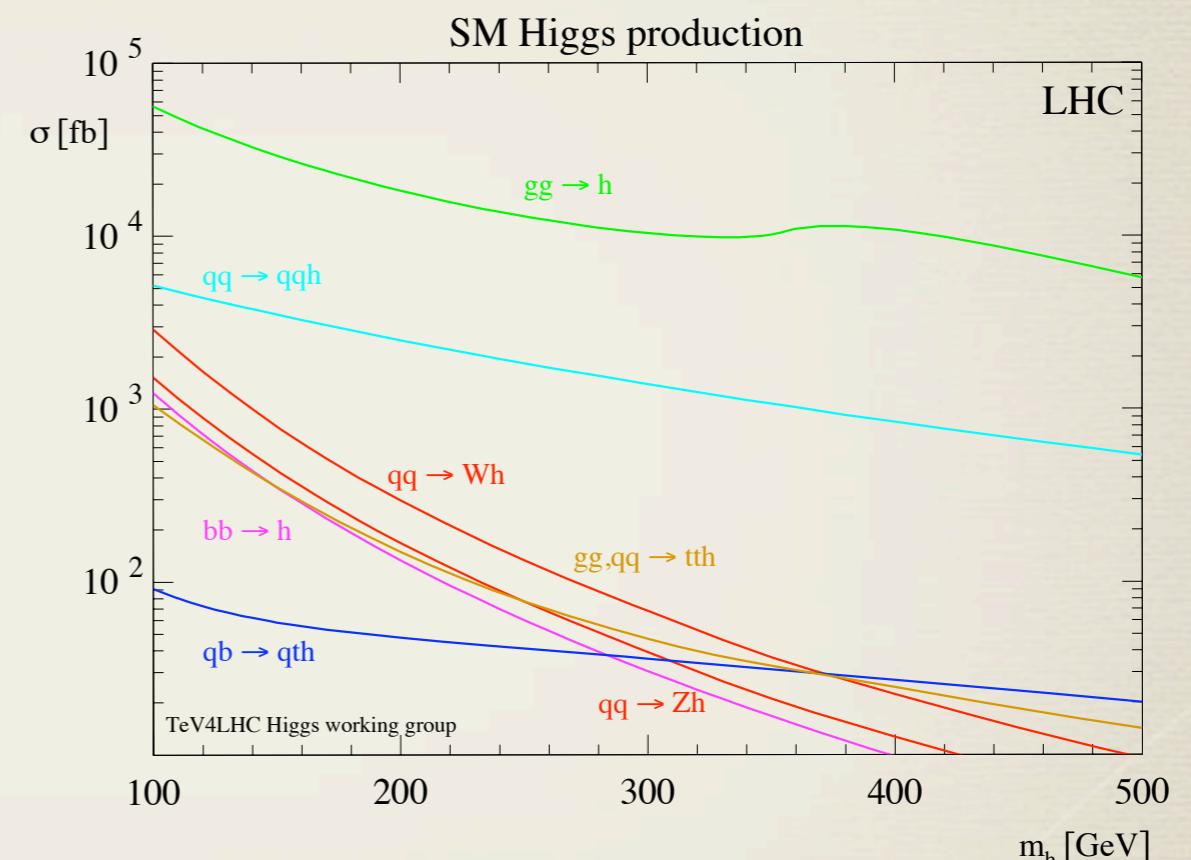
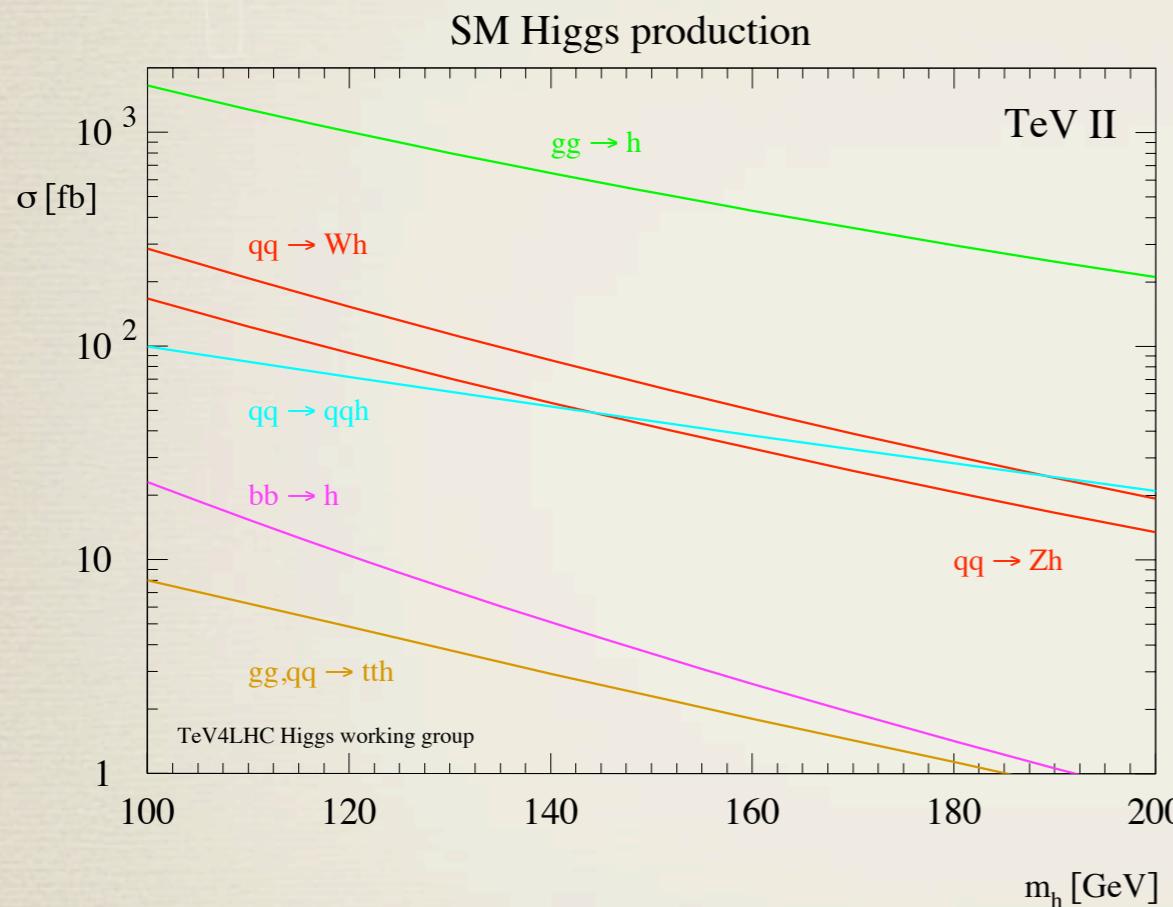
Can't do LEP search, \sqrt{s} not fixed at hadron machine



Any hadron collider search must confront backgrounds

Overview of Higgs cross sections

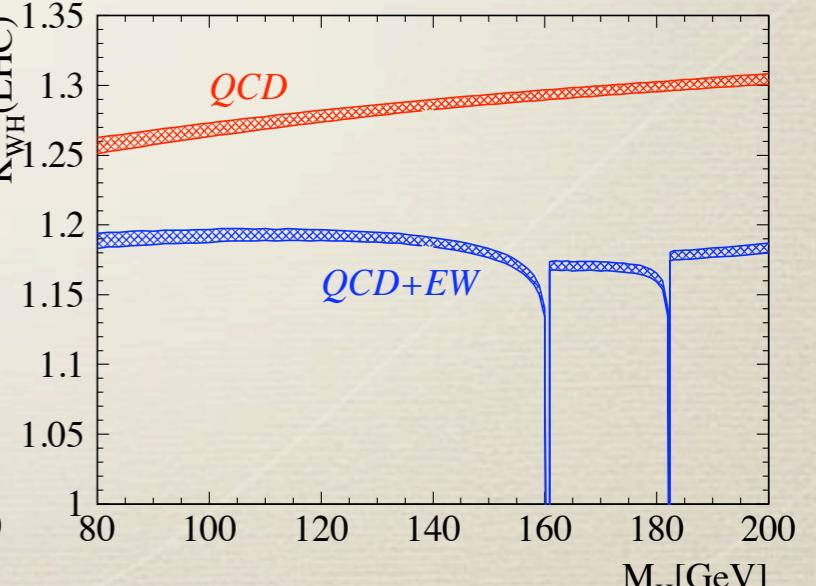
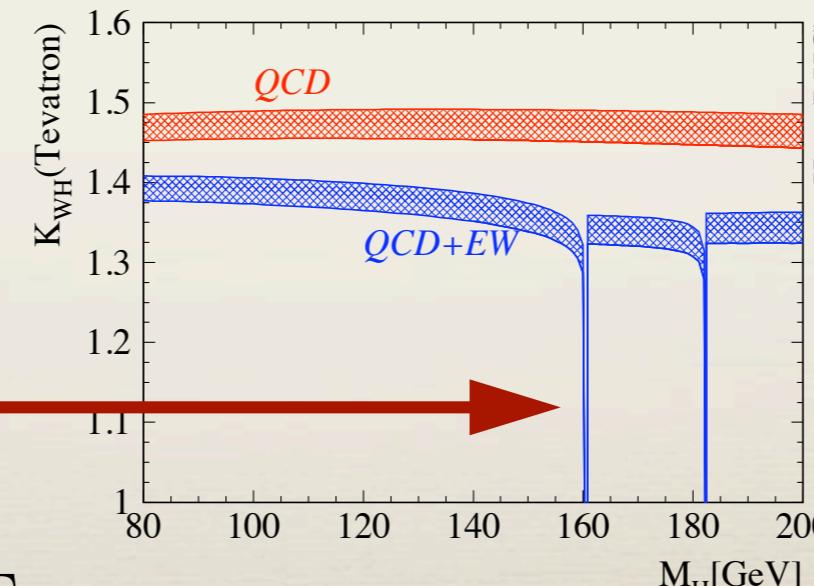
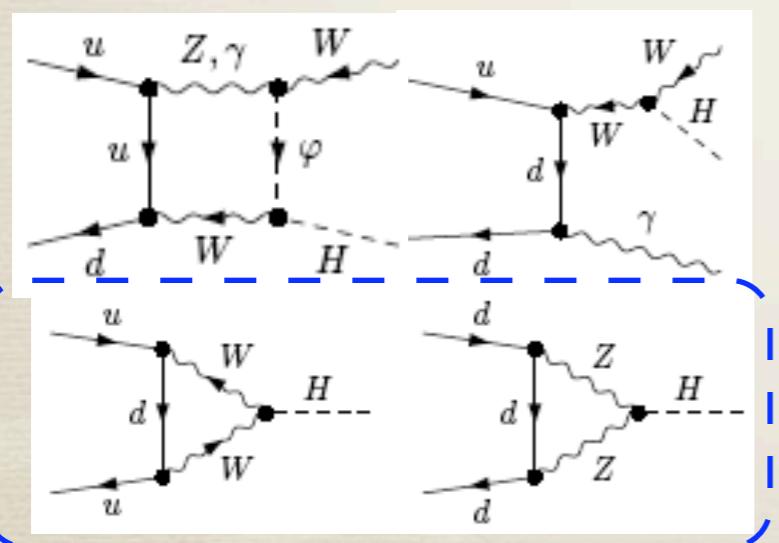
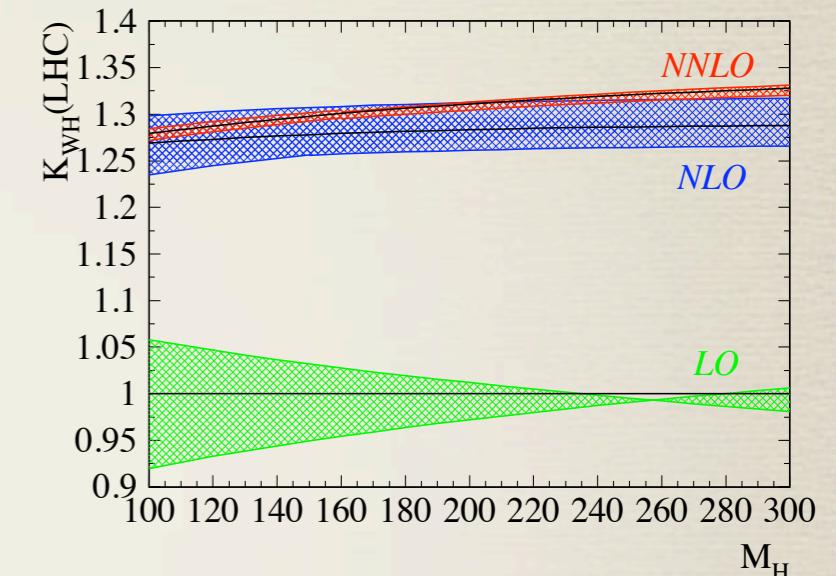
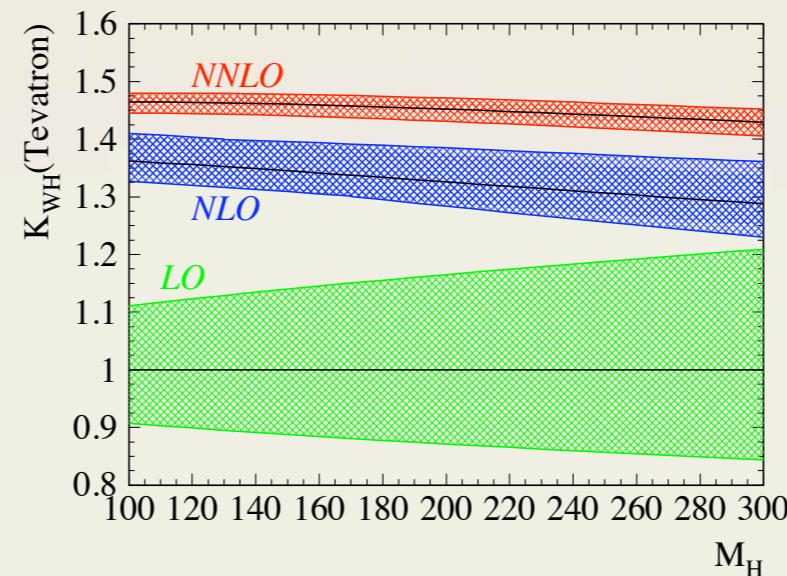
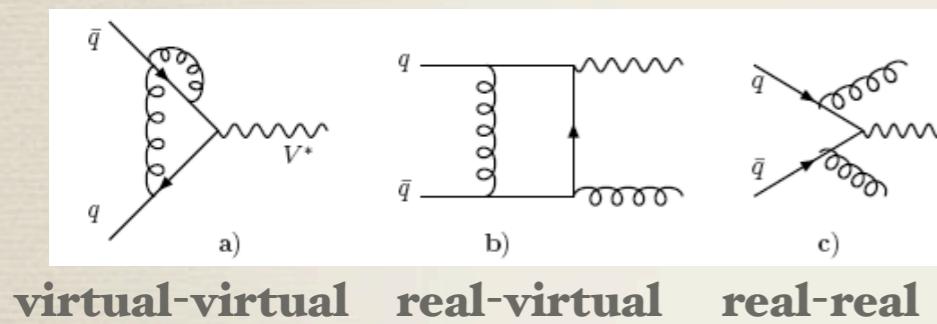
- * Gluon-fusion dominant at both colliders; WH next at Tevatron, WBF at LHC



- * SUSY: $g_{bbh} \sim \tan^2 \beta$; becomes dominant at $\tan \beta \sim 10$
- * Plan: discuss signal cross section properties, then move on to searches at Tevatron/LHC

W/Z associated production

- * Leptonic W, Z decays offer experimental handle, especially when $h \rightarrow b\bar{b}$ for low M_H

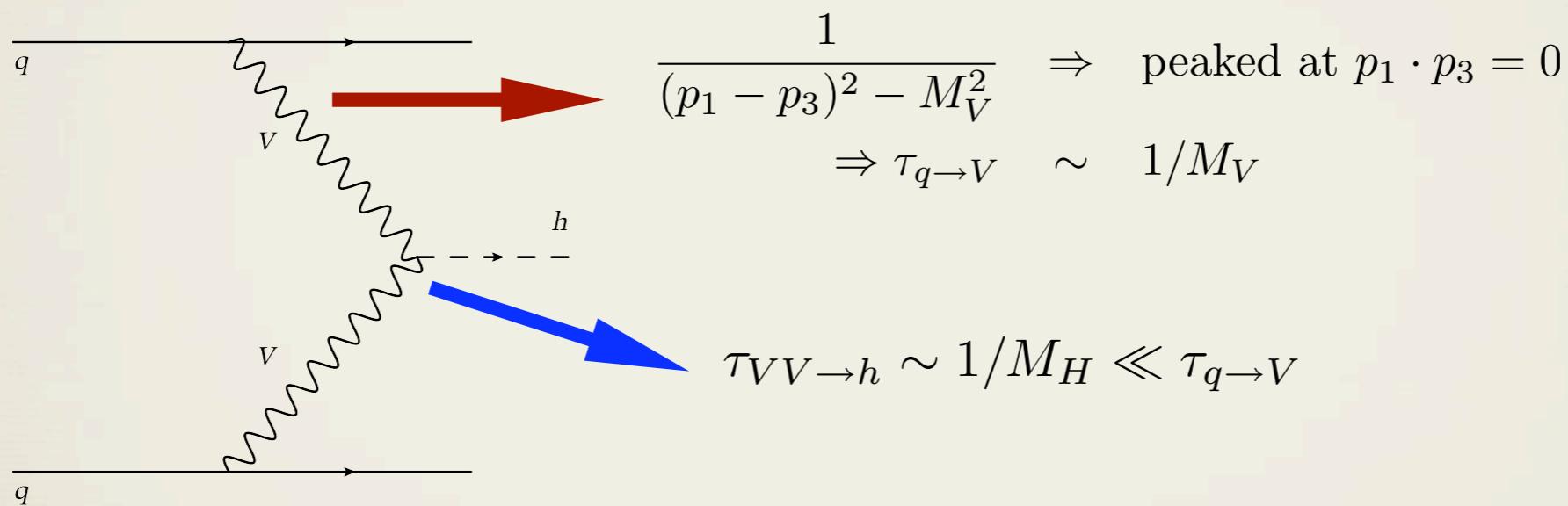


Replace $M_V^2 \rightarrow M_V^2 - iM_V\Gamma_V$

Brein et al., hep-ph/0402003

Weak boson fusion: effective W/Z

- * Important throughout large region of Higgs mass and in many decay modes; forward jets give experimental handle
- * First approximation: inclusive cross section for $M_H \gg M_{W,Z}$



- * Should be able to factorize, think of V as a parton in q

$$\sigma_{q\bar{q} \rightarrow VV \rightarrow hh} = \int dz_1 dz_2 f_{q/V_1}(z_1) f_{q/V_2}(z_2) \sigma_{VV \rightarrow hh}$$

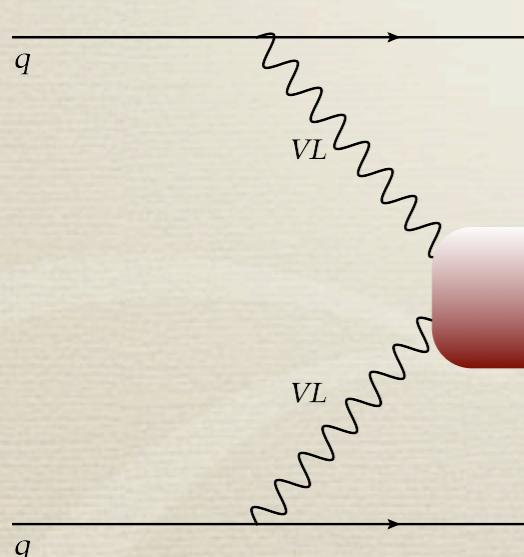
WBF + the equivalence theorem

- * Can derive when $M_V \ll \sqrt{s}$ (small angle scattering dominated)

$$\begin{aligned}\sigma_{q_1 q_2 \rightarrow VV \rightarrow h} &= \int_{2M_V/\sqrt{\hat{s}}}^1 dz_1 \int_{2M_V/\sqrt{\hat{s}}}^1 dz_2 f_{q/V_L}(z_1) f_{q/V_L}(z_2) \sigma_{V_L V_L \rightarrow h}(z_1 z_2 \hat{s}) \\ \sigma_{V_L V_L \rightarrow h}(x) &= \frac{\pi}{36} g_{HVV}^2 \frac{x}{M_V^2} \delta(x - M_H^2) \\ f_{q/V_L}(z) &= \frac{g_v^2 + g_a^2}{4\pi^2} \frac{1-z}{z}\end{aligned}$$

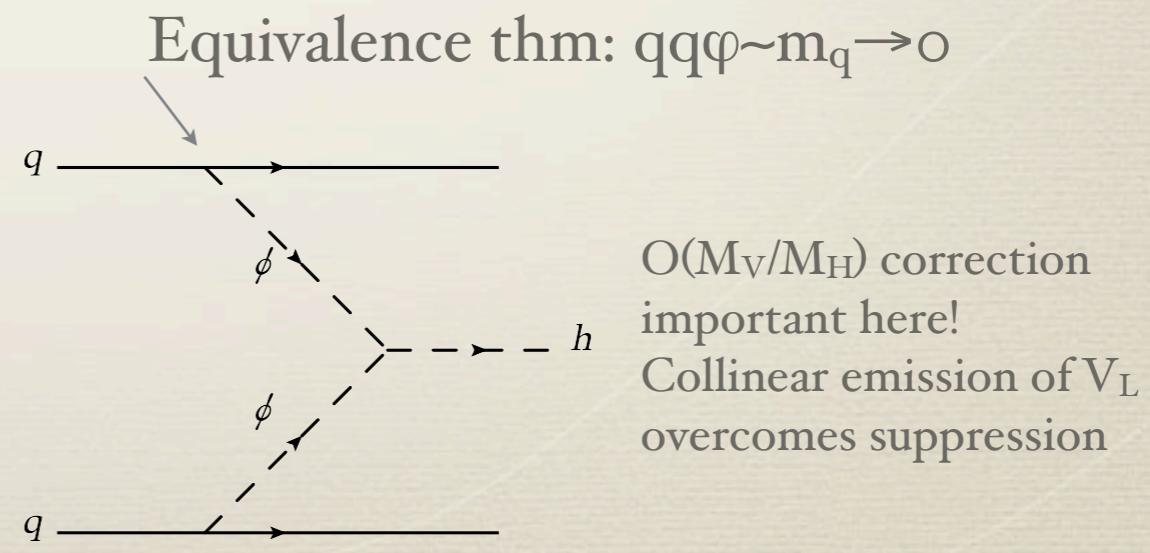
Derive this

- * Angular momentum cons. prevents emission of transverse boson with forward quark: $\bar{u}^\pm(p\hat{z}) \not\epsilon u^\pm(p'\hat{z}) \Rightarrow$ Set $\not{e} = \gamma^{1,2} \Rightarrow \xi_\pm^\dagger \sigma^{1,2} \xi_\pm = 0$



Good channel to study strong EWSB

?

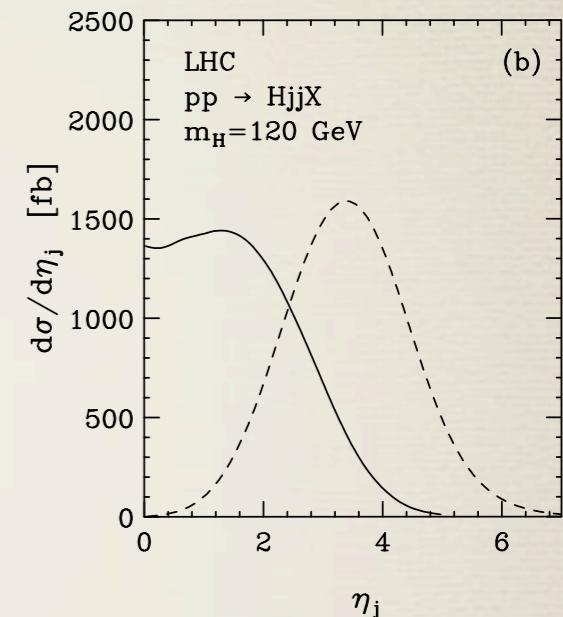
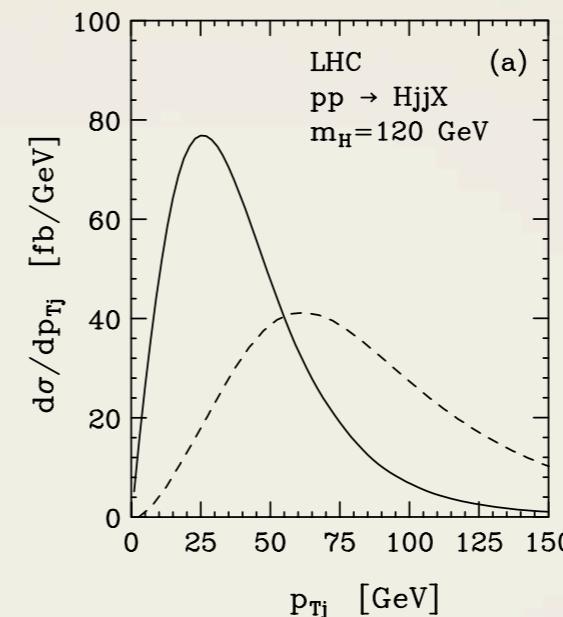
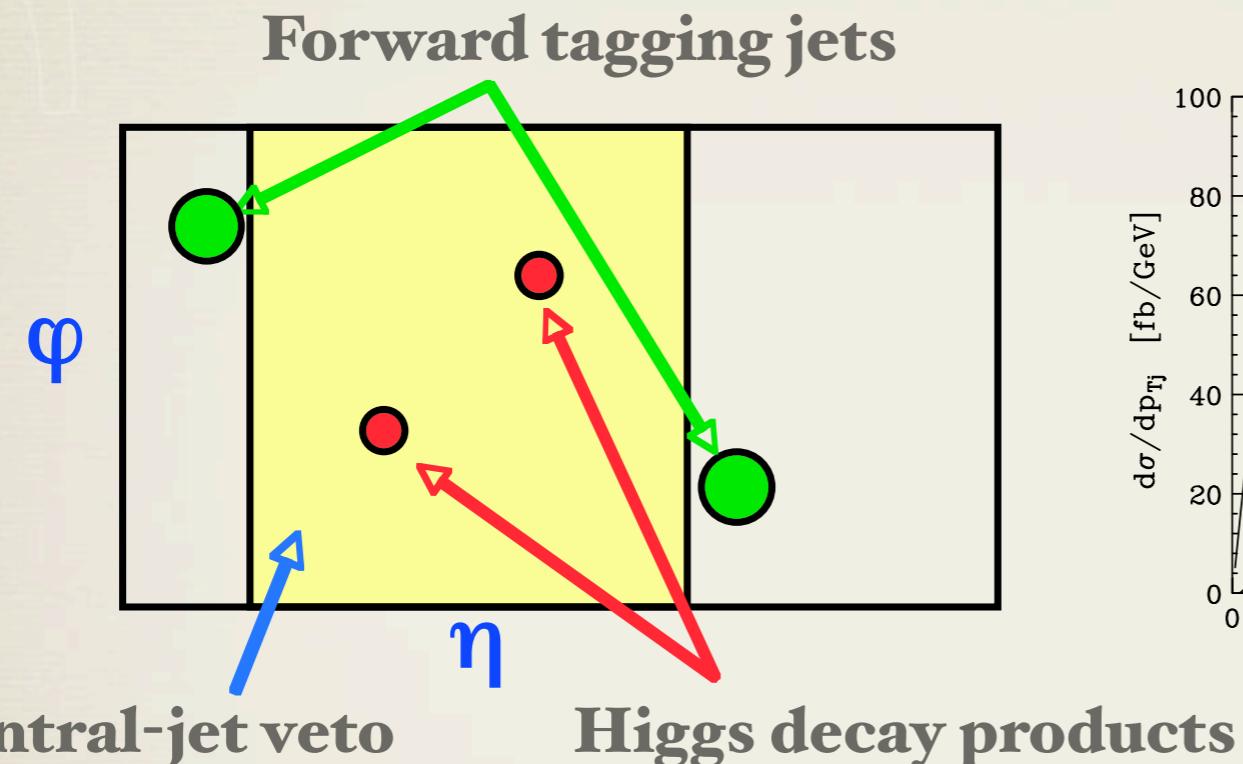


$O(M_V/M_H)$ correction important here!
Collinear emission of V_L overcomes suppression

Dawson 1984; Chanowitz, Gaillard 1985

Kinematics of WBF

- * Two energetic ($p_T \sim 40$ GeV) jets with large rapidity separation



Rainwater, Zeppenfeld hep-ph/9906218
and many others... check refs+citations

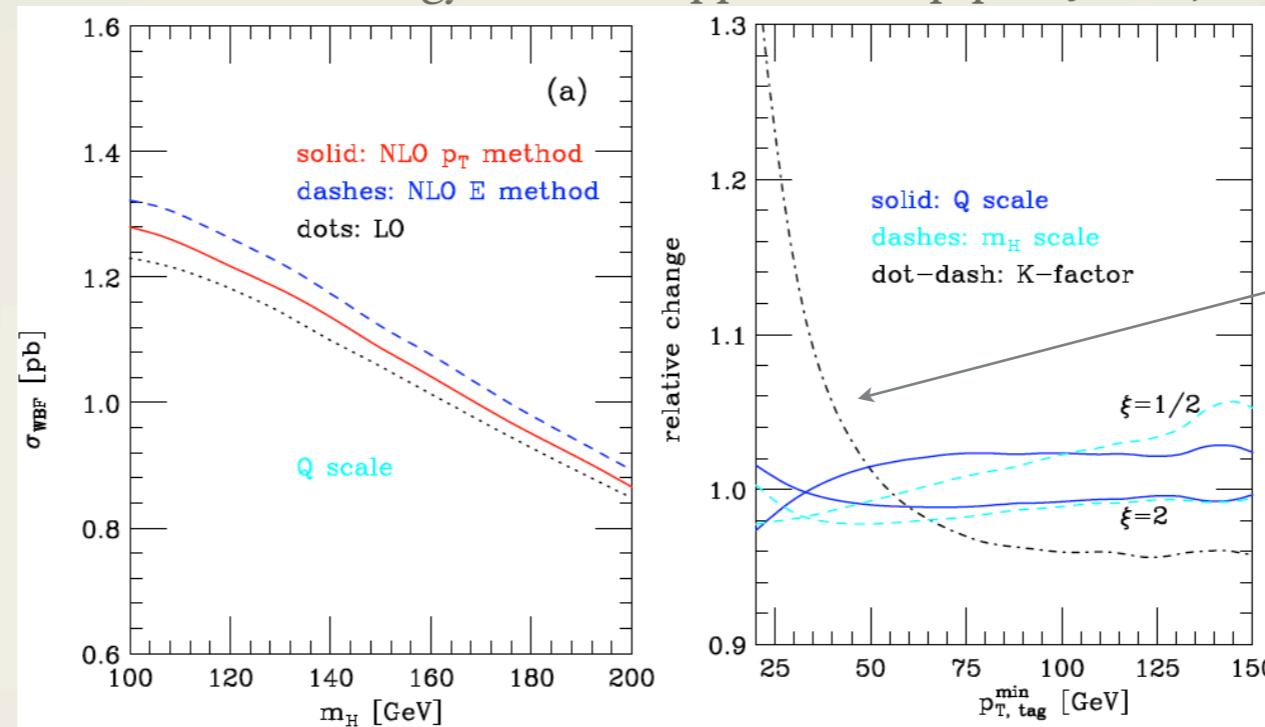
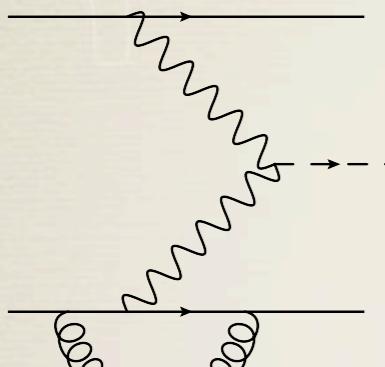
- * Extra gluon emission suppressed; impose central jet veto

$$\begin{aligned} \mathcal{M}(q_1 q_2 \rightarrow q_3 q_4 h + g) &\propto \mathcal{M}(q_1 q_2 \rightarrow q_3 q_4 h) T^a \left\{ \frac{p_3 \cdot \epsilon_g^a}{p_3 \cdot p_g} + \frac{p_4 \cdot \epsilon_g^a}{p_4 \cdot p_g} - \frac{p_1 \cdot \epsilon_g^a}{p_1 \cdot p_g} - \frac{p_2 \cdot \epsilon_g^a}{p_2 \cdot p_g} \right\} \\ &\rightarrow 0 \text{ since } p_1 \parallel p_3, \quad p_2 \parallel p_4 \end{aligned}$$

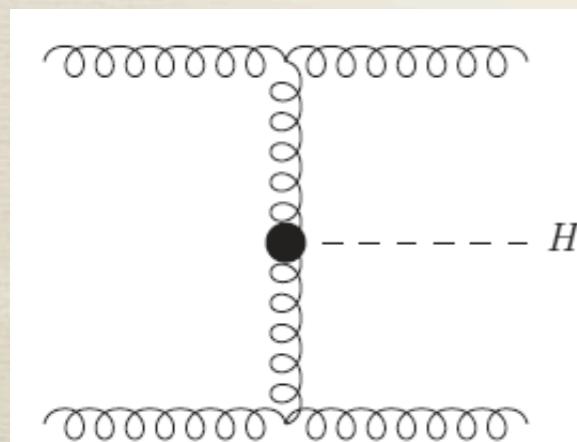
QCD corrections to WBF

- * Pentagons give $\text{Tr}[T^a] \times \text{Tr}[T^b] = 0$; only vertex corrections

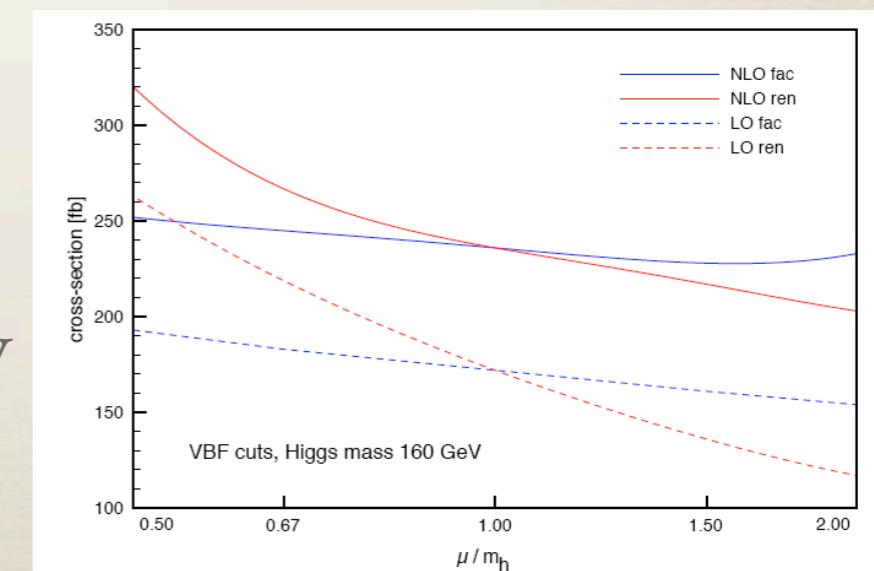
Figy, Oleari, Zeppenfeld hep-ph/0306109



Corrections have kinematic dependence



Passes central-jet veto, but under control theoretically



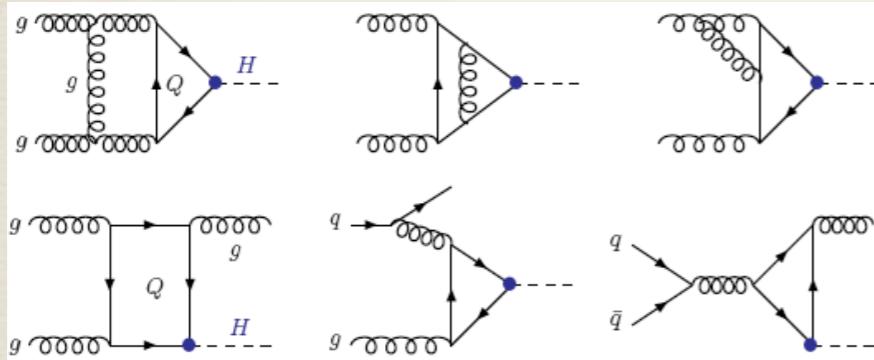
Campbell, Ellis, Zanderighi hep-ph/0608194

Gluon fusion production

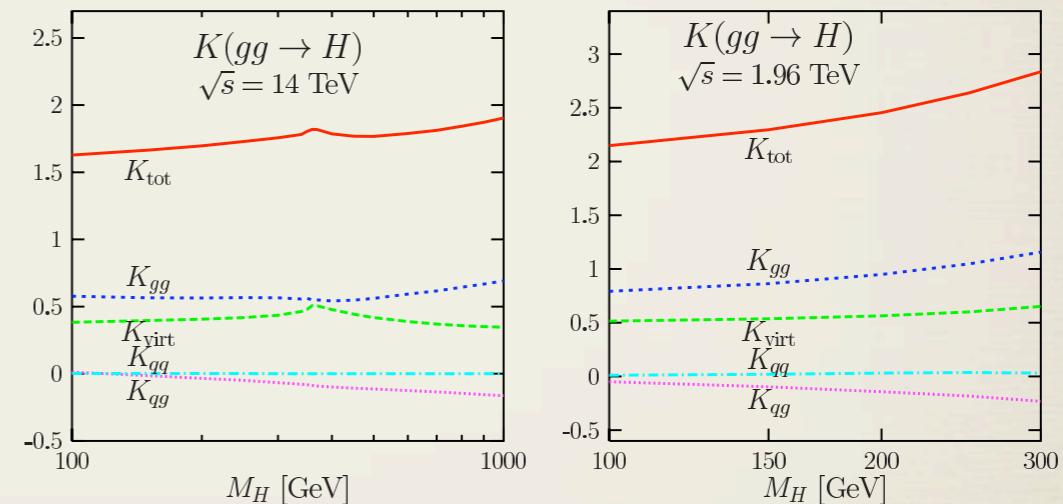
- * Largest mode at Tevatron and LHC; through top-quark loops

$$\sigma_{gg \rightarrow h}^{LO} = \frac{G_F \alpha_s^2}{288\pi\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \delta(1-z), \quad \tau_Q = \frac{M_H^2}{4m_Q^2}, \quad z = \frac{M_H^2}{\hat{s}}$$

- * NLO QCD corrections require 2-loop virtual, 1-loop real-virtual



Djouadi, Graudenz, Spira, Zerwas PLB 264 (1991), hep-ph/9504378



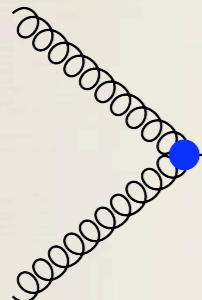
- * Can reach $K_{NLO} = \sigma_{NLO}/\sigma_{LO} \approx 2$ at LHC, 3 at Tevatron; why so large?

A case study in QCD: gluon fusion

EFT gluon-fusion

- * Study carefully in the EFT with Hgg vertex

$$\sigma_{ij} = \sigma_0 \left\{ G_{ij}^{(0)} + \frac{\alpha_s}{\pi} G_{ij}^{(1)} + \mathcal{O}(\alpha_s^2) \right\} \quad \mathcal{L}_{EFT} = \frac{1}{12} \frac{\alpha_s}{\pi} \left[1 + \frac{11}{4} \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right]$$



$$G_{ij}^{(0)} = \delta(1-z) \delta_{ig} \delta_{jg} \quad (\text{Overall factors in } \sigma_0)$$

($z = M^2/x_1/x_2/s$; integral over PDFs \Rightarrow integral over z)

- * NLO receives contributions from 5 pieces: virtual diagrams, real-radiation, ultraviolet renormalization, PDF renormalization correction to EFT coefficient

- Everything in $d=4-2\epsilon$ dimensions;
gluon has $d-2$ polarization states

- Scaleless integrals vanish

- Coupling constant gets dimensions:

$$g \rightarrow g \mu^\epsilon$$

- Feynman gauge

- Only gg initial-state (others smaller, simpler)

$$\int d^d k \frac{1}{k^2} \sim \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}} \rightarrow 0$$

J. Collins, *Renormalization*

Notes:

Gluon-fusion: virtual

* Virtual:

$$= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left(\frac{\mu^2}{\hat{s}} \right)^\epsilon \left\{ -\frac{13}{4\epsilon} - \frac{83}{12} \right\} \delta(1-z)$$

$$= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left(\frac{\mu^2}{\hat{s}} \right)^\epsilon \left\{ \frac{3}{\epsilon^2} + \frac{1}{4\epsilon} + \frac{47}{12} + 2\pi^2 \right\} \delta(1-z)$$

Leading soft+collinear singularity; emitting gluons from gluons gives color factor $C_A=3$

* UV renormalization: counterterm for α_s at leading order

Full d-dimensional LO

$$= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \frac{1}{\epsilon} G_{gg}^{(0),d} \{-2 b_0\}$$

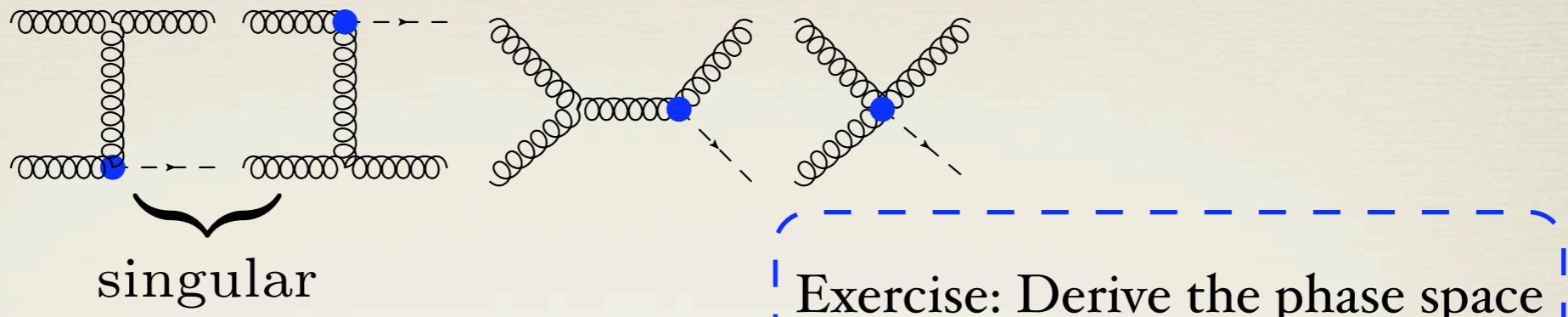
First term in beta-function

$$= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left\{ -\frac{11}{2} + \frac{N_F}{3} \right\} \left[\frac{1}{\epsilon} + 1 \right] \delta(1-z)$$

Number of light fermions

Gluon-fusion: real radiation

* Real:



$$\begin{aligned}
\text{Phase space} & : \frac{1}{2\hat{s}} \int \frac{d^d p_g}{(2\pi)^d} \frac{d^d p_H}{(2\pi)^d} (2\pi) \delta(p_g^2) (2\pi) \delta(p_H^2 - M_H^2) (2\pi)^d \delta^{(d)}(p_1 + p_1 - p_g - p_H) \\
& = \frac{1}{16\pi\hat{s}} \frac{s^{-\epsilon}}{(4\pi)^{-\epsilon} \Gamma(1-\epsilon)} (1-z)^{1-2\epsilon} \int_0^1 d\lambda \lambda^{-\epsilon} (1-\lambda)^{-\epsilon} \\
& \Rightarrow \hat{t} = (p_1 - p_q)^2 = -\hat{s}(1-z)\lambda, \quad \hat{u} = (p_2 - p_q)^2 = -\hat{s}(1-z)(1-\lambda)
\end{aligned}$$

Real radiation and plus dists.

- * Extract singularities using plus distribution expansion

$$\lambda^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(\lambda) + \frac{1}{[\lambda]_+} - \epsilon \left[\frac{\ln \lambda}{\lambda} \right]_+ + \mathcal{O}(\epsilon^2), \text{ etc.}$$

$$\int_0^1 dx f(x)[g(x)]_+ = \int_0^1 dx [f(x) - f(0)] g(x)$$

$$\Rightarrow \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left(\frac{\mu^2}{\hat{s}} \right)^\epsilon \left\{ \overbrace{\left[\frac{3}{\epsilon^2} + \frac{3}{\epsilon} \right] \delta(1-z)}^{\text{cancels virtual poles}} - \frac{6}{\epsilon} \frac{1}{[1-z]_+} + \frac{6z(z^2 - z + 2)}{\epsilon} \right. \\ \left. + (3 - \cancel{\pi^2}) \delta(1-z) - \frac{6}{[1-z]_+} + \cancel{12} \left[\frac{\ln(1-z)}{1-z} \right]_+ - \frac{6(z^2 - z + 1)^2 \ln z}{1-z} \right. \\ \left. - 12z(z^2 - z + 2) \ln(1-z) - \frac{11}{2} + \frac{57z}{2} - \frac{45z^2}{2} + \frac{23z^3}{2} \right\}$$

Remaining terms

- * PDF renormalization: counterterm for initial-state collinear sing.

$$\begin{aligned}
 &= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \frac{1}{\epsilon} \textcolor{blue}{G_{gg}^{(0),d}}(\textcolor{red}{\bar{P}_{gg}}(\textcolor{red}{z})) \longrightarrow \text{Altarelli-Parisi splitting function} \\
 &= \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left\{ \underbrace{\left(\frac{11}{2} - \frac{N_F}{3} \right) \delta(1-z)}_{\text{cancels UV counterterm}} + \underbrace{\frac{6}{[1-z]_+} - 6z(z^2 - z + 2)}_{\text{cancels real radiation}} \right\} \left[\frac{1}{\epsilon} + 1 \right]
 \end{aligned}$$

- * Coefficient correction: $= \frac{\alpha_s}{\pi} \frac{11}{2} \delta(1-z)$
- * Can check that $(\mu^2/s)^\epsilon$ terms give $\ln(\mu^2/s)$ upon expansion \Rightarrow combined with scale dependence of α_s (implicit so far) and PDFs give estimate of theoretical uncertainty (can also get these logs from renormalization group considerations)

Final result and π^2

- * Final result for NLO correction:

$$\boxed{G_{gg}^{(1)} = \left(\frac{11}{2} + \pi^2\right) \delta(1-z) + 12 \left[\frac{\ln(1-z)}{1-z} \right]_+ - 12z(-z+z^2+2)\ln(1-z) - 6 \frac{(z^2+1-z)^2}{1-z} \ln(z) - \frac{11}{2}(1-z)^3}$$

(M²/s ≤ z ≤ 1)
 (integrate over
PDFs, removes
singularity)

- * What is the source of the π^2 ? Since $1/\epsilon^2$ poles cancel, can change that $\Gamma(1+\epsilon)$ normalizing the real, virtual can be exchanged for something that differs at $O(\epsilon^2) \Rightarrow$ shuffles terms between R, V

$$\Gamma(1+\epsilon) \rightarrow \underbrace{\frac{1}{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}}_{\mathcal{N}} \Gamma^2(1-\epsilon)\Gamma^2(1+\epsilon) = \mathcal{N} \left(1 + \frac{\pi^2}{3}\epsilon^2 + \mathcal{O}(\epsilon^4) \right)$$

Real: $\mathcal{N} \left(1 + \frac{\pi^2}{3}\epsilon^2 \right) \left(\frac{3}{\epsilon^2} - \pi^2 + \dots \right) \delta(1-z) = \mathcal{N} \left(\frac{3}{\epsilon^2} + \dots \right)$

Virtual: $\mathcal{N} \left(1 + \frac{\pi^2}{3}\epsilon^2 \right) \left(-\frac{3}{\epsilon^2} + 2\pi^2 + \dots \right) \delta(1-z) = \mathcal{N} \left(-\frac{3}{\epsilon^2} + \pi^2 + \dots \right)$

Completely from analytic continuation: $6(-\mu^2/s)^\epsilon = (\mu^2/s)^\epsilon \times (6 + \pi^2 + \text{imaginary parts} + \dots)$

From $C_A=3$ color, $2 \times \text{Re}(M_o M_I^*)$

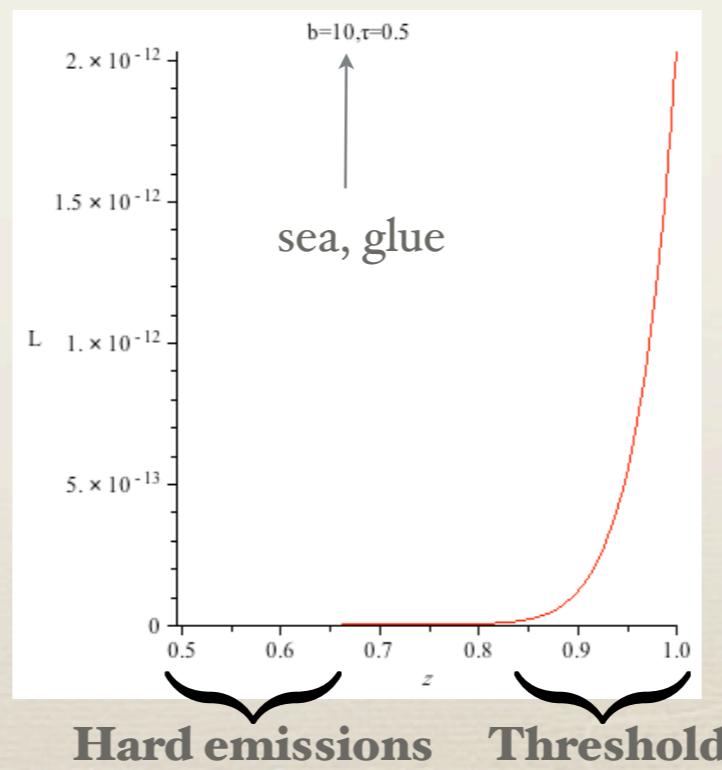
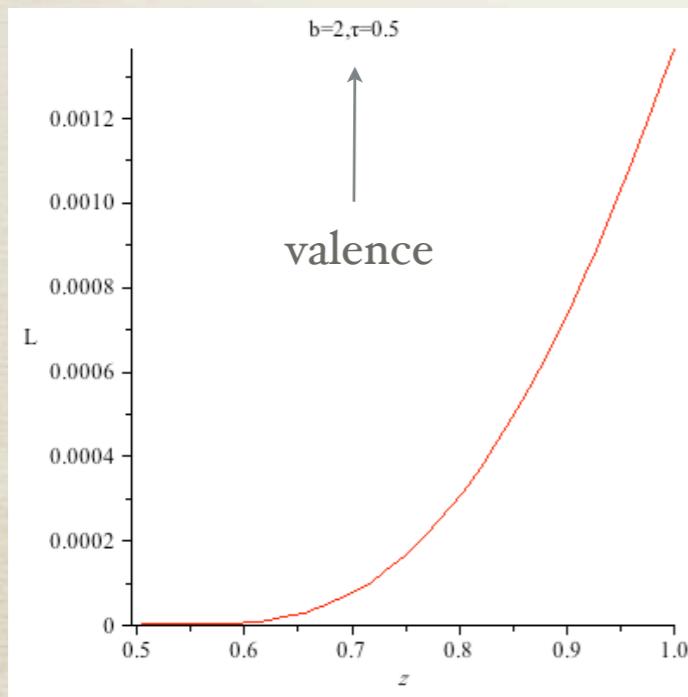
Threshold logs and PDFs

- * Logarithm is associated with soft radiation; is $z \rightarrow 1$ region enhanced?
- * Begin with hadronic cross section in following form ($\tau = M^2/s$, $z = M^2/x_1 x_2 s$)

$$\sigma_{had} = \tau \int_{\tau}^1 dz \frac{\sigma(z)}{z} \mathcal{L}\left(\frac{\tau}{z}\right), \quad \overbrace{\mathcal{L}(y) = \int_y^1 dx \frac{y}{x} f_1(x) f_2(y/x)}^{\text{partonic luminosity}}$$

Assume $f_i \sim x^a(1-x)^b$; plot L for various τ, b (' a ' less important)

Look for peak near $z \approx 1$

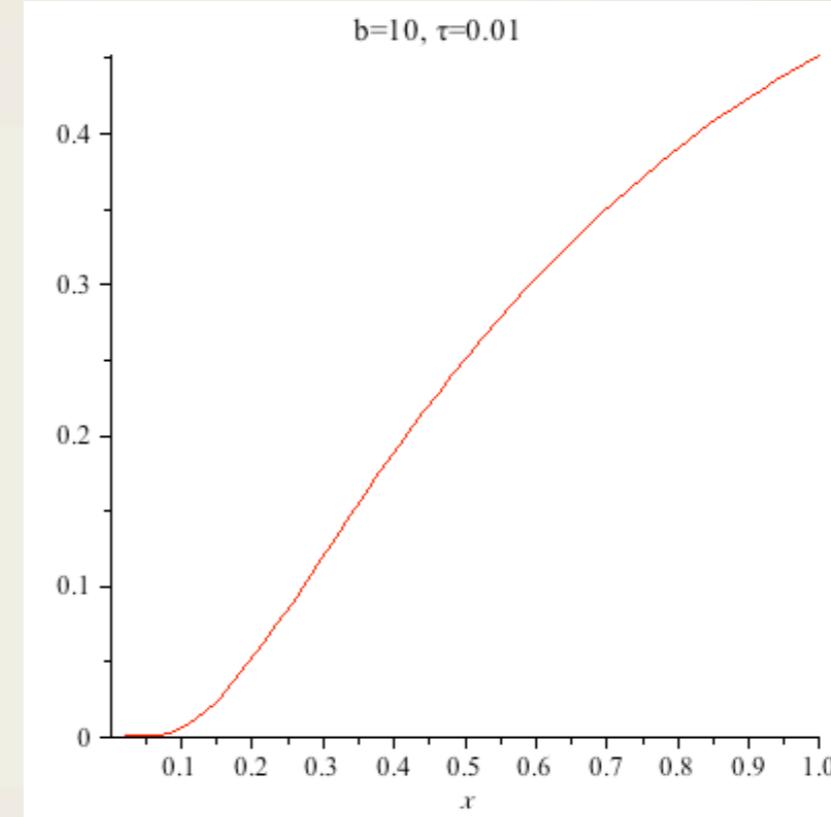
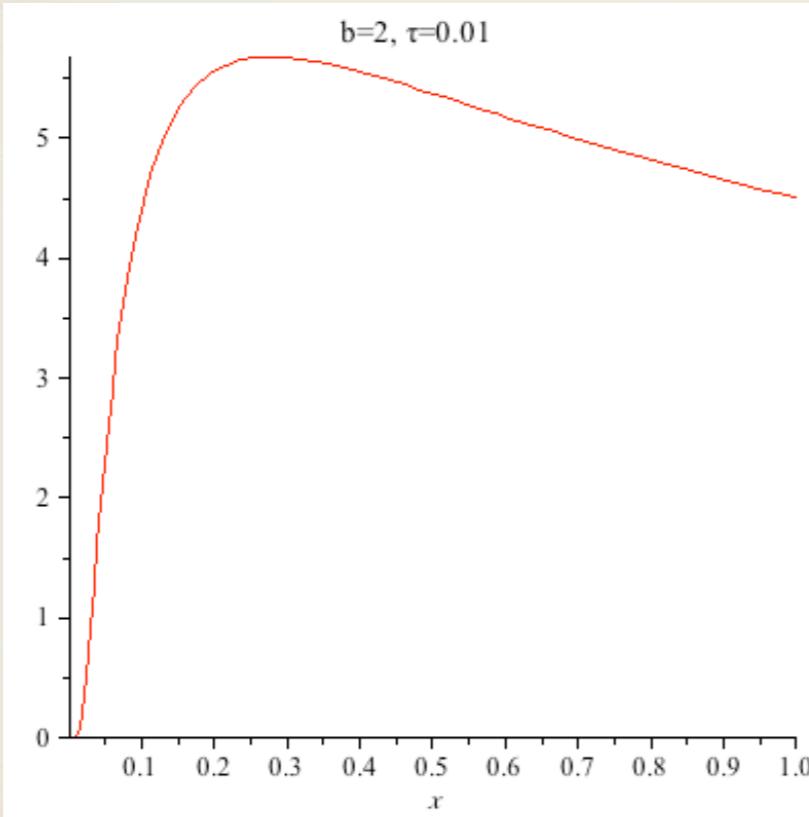


Clear importance for $\tau \approx 1$; rapid fall-off of large- z PDFs

$$\int_0^1 dx \left[\frac{\ln^n(1-x)}{1-x} \right]_+ \overbrace{\theta(x - x_{cut})}^{\text{approximate } \mathcal{L}/x} = -\frac{1}{n+1} \ln^{n+1}(1 - x_{cut})$$

Threshold logs and PDFs

- * Shape of PDFs near $z=1$ important for small τ ; large exponents in $(1-z)^b$ can enhance region where logs are large



- * A numerical question regarding how dominant the logarithmic terms are in the perturbative expansion; for Higgs, both plus dist. and $\delta(1-z)$ give large corrections

(discussions in Kramer, Laenen, Spira hep-ph/9611272, Catani et al. hep-ph/0306211, Ahrens, Becher, Neubert, Yang 0809.4283)

Effective theory validity

- * Clearly want to go beyond NLO, but the 3-loop massive computations in the full theory are intractable

$$\sigma_{NLO}^{approx} = \left(\frac{\sigma_{NLO}^{EFT}}{\sigma_{LO}^{EFT}} \right) \sigma_{LO}^{QCD}$$

%-level or better for $M_H < 2m_t$, even
gets >90% of correction above

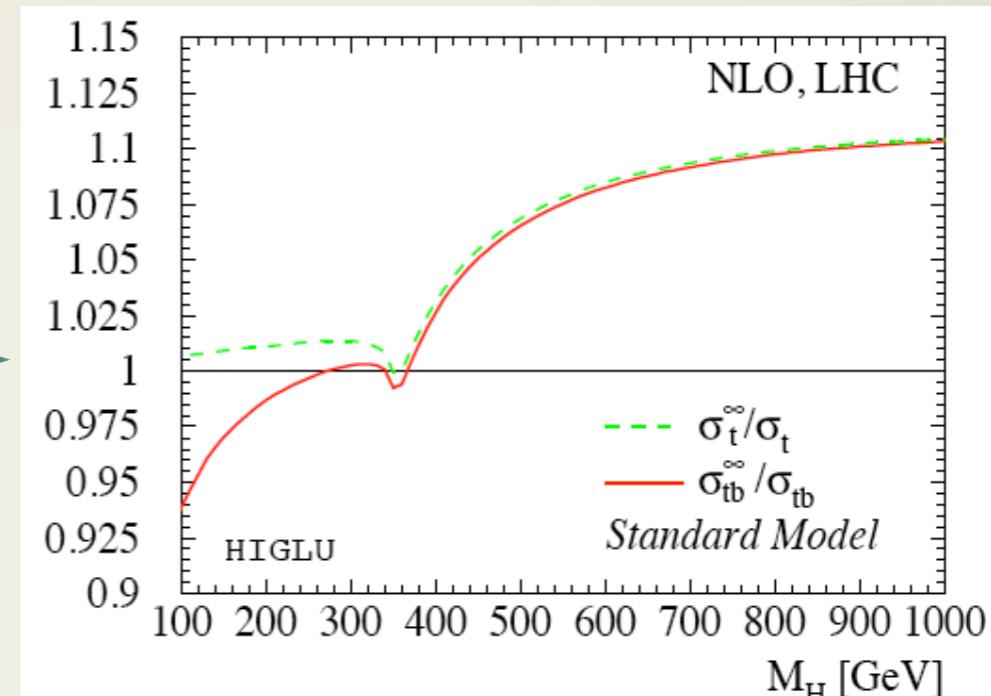


Soft, collinear gluons do not resolve top-quark loop (e.g., soft gluons are eikonal×tree)

Kramer, Laenen, Spira; Marzani et al. 0801.2544

⇒ same in full theory, EFT

- * Use EFT to go to NNLO

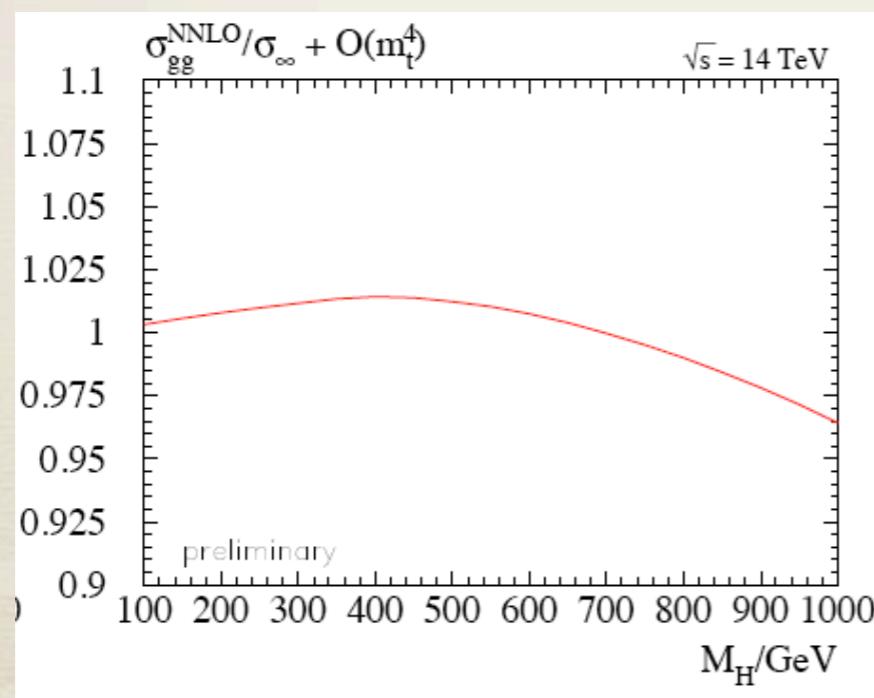
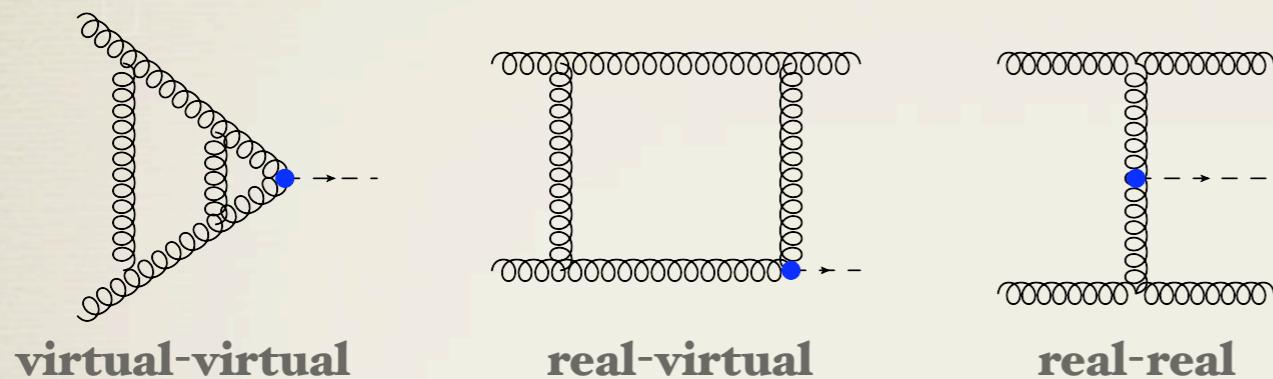


Harlander, 2009 Zurich Higgs workshop

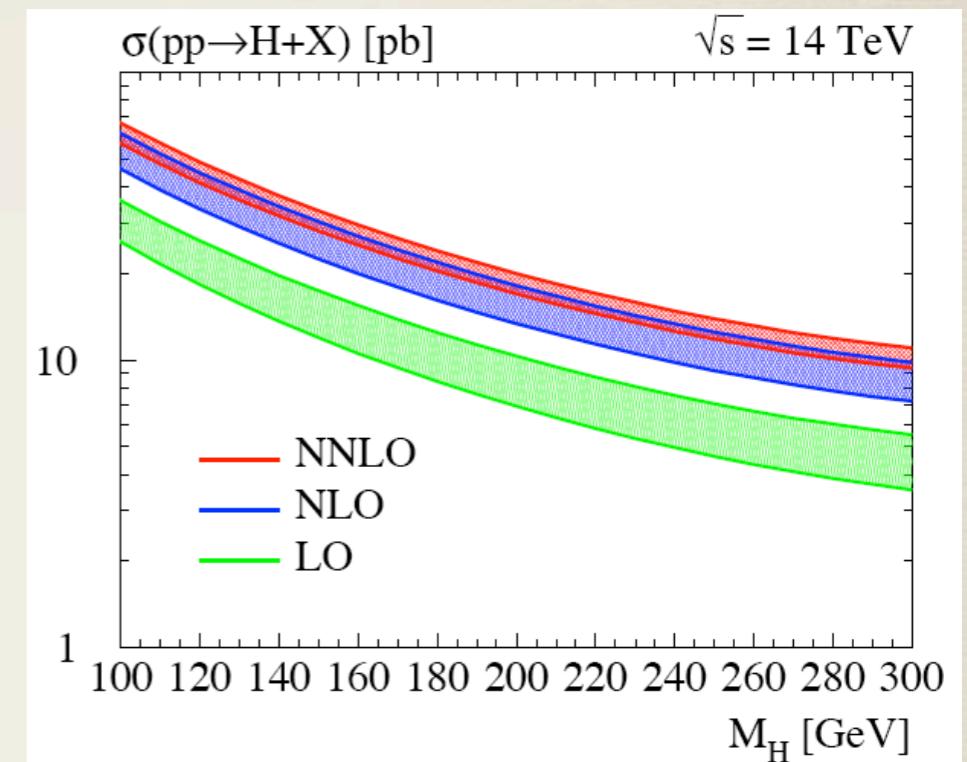
$$\sigma_{NNLO}^{approx} = \left(\frac{\sigma_{NNLO}^{EFT}}{\sigma_{LO}^{EFT}} \right) \sigma_{LO}^{QCD}$$

Inclusive Higgs at NNLO

- * Full calculation at NNLO in the EFT



K. Ozeren (w/ R. Harlander), LoopFest 2009

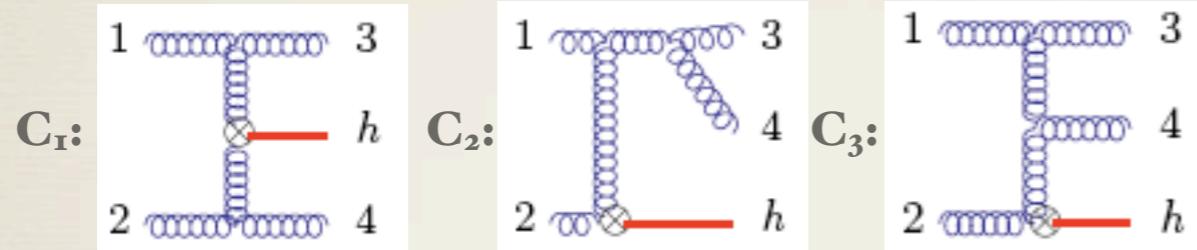


Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith van Neerven '03

Recent asymptotic expansion of sub-leading $1/m_t$ terms at NNLO indicates they are small

Exclusive production at NNLO

- * Exclusive results needed to check affect of experimental cuts
- * Challenge: Describe singular limits of double real emission without inclusive integration



p_3, p_4 can become soft/collinear

Explicit choice of phase-space parameterizations as at NLO:

$s_{13}, s_{14}, s_{23}, s_{24} \rightarrow 0$ limits take factorized form; expand each λ_i separately in plus distributions; useful for $C_1 \rightarrow$ diagrams

$$\begin{aligned} d\Pi_E &= \frac{1}{4} \int dE_3 dE_4 d\Omega_3 d\Omega_4 E_3^{d-3} E_4^{d-3} \delta [1 - z + s_{13} + s_{14} + s_{23} + s_{24} + s_{34}] \\ d\Pi_E &= N \int_0^1 d\lambda_1 d\lambda_2 d\lambda_3 d\lambda_4 [\lambda_1(1-\lambda_1)]^{1-2\epsilon} [\lambda_2(1-\lambda_2)]^{-\epsilon} [\lambda_3(1-\lambda_3)]^{-\epsilon} \\ &\quad \times [\lambda_4(1-\lambda_4)]^{-\epsilon-1/2} D^{2-d}, \\ s_{13} &= -(1-z)\lambda_1(1-\lambda_2), \quad s_{23} = -(1-z)\lambda_1\lambda_2, \\ s_{14} &= -(1-z)(1-\lambda_1)(1-\lambda_3)/D, \quad s_{24} = -(1-z)(1-\lambda_1)\lambda_3/D \end{aligned}$$

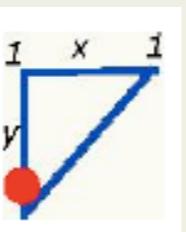
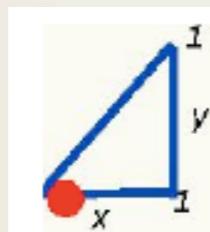
Alternate parameterizations can be chosen to factorize C_2, C_3 singularities

Sector decomposition

- * Difficulties with $C_i \times C_j$, $i \neq j$; no choice factorizes all singularities

$$\begin{aligned}\mathcal{I} &= \int_0^1 dx dy \frac{x^\epsilon y^\epsilon}{(x+y)^2} \\ \Rightarrow \mathcal{I}_{naive} &= \int_0^1 dx dy \frac{1}{(x+y)^2} \{1 + \mathcal{O}(\epsilon)\} \\ &= \int_0^1 dy \frac{1}{y(1+y)} + \mathcal{O}(\epsilon)\end{aligned}$$

Must first define
an ordering for x,y:



$$\mathcal{I} = \int_0^1 dx \int_0^x dy \frac{x^\epsilon y^\epsilon}{(x+y)^2} + \int_0^1 dy \int_0^y dx \frac{x^\epsilon y^\epsilon}{(x+y)^2}$$

Now rescale integration
regions back to unit square:

$$\mathcal{I} = \int_0^1 dx dy \frac{x^{-1+2\epsilon} y^\epsilon}{(1+y)^2} + \int_0^1 dx dy \frac{y^{-1+2\epsilon} x^\epsilon}{(1+x)^2}$$

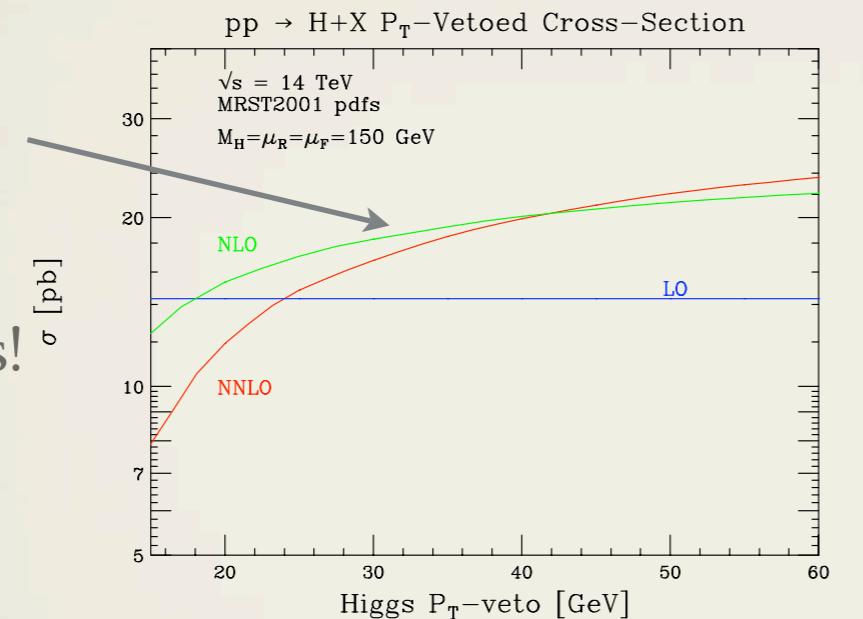
Recursive application factorizes all singularities in double-real emission

Exclusive production at NNLO

- * Codes exist that simulate fully differential Higgs production

(FEHiP: Anastasiou, Melnikov, FP hep-ph/0409088, 0501130; HNNLO: Catani, Grazzini hep-ph/0703012, 0802.1410)

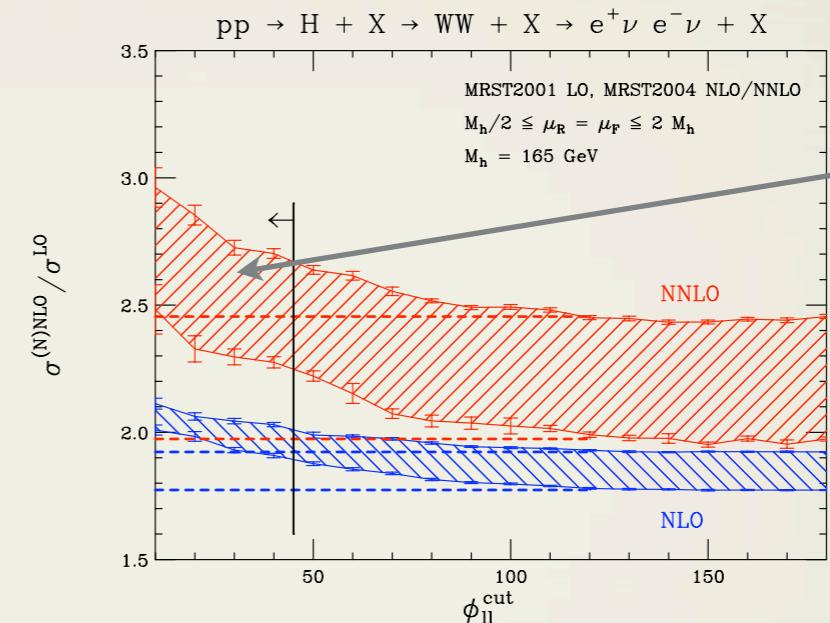
With jet-veto,
NLO>NNLO;
hard emissions
important for
detailed studies!
Not just
threshold, π^2 !



Anastasiou, Melnikov, FP hep-ph/0501130

p_T^{veto} (GeV)	σ_{NLO} (fb)	σ_{NNLO} (fb)
no veto	21.26 ± 0.05	22.21 ± 0.32
40	18.62 ± 0.05	17.38 ± 0.34
30	17.18 ± 0.05	15.74 ± 0.35
20	14.42 ± 0.05	11.31 ± 0.38

Catani, Grazzini 0802.140



Anastasiou, Dissertori, Stoeckli 0707.2373

Not the
inclusive K-
factor in the
WW signal
region

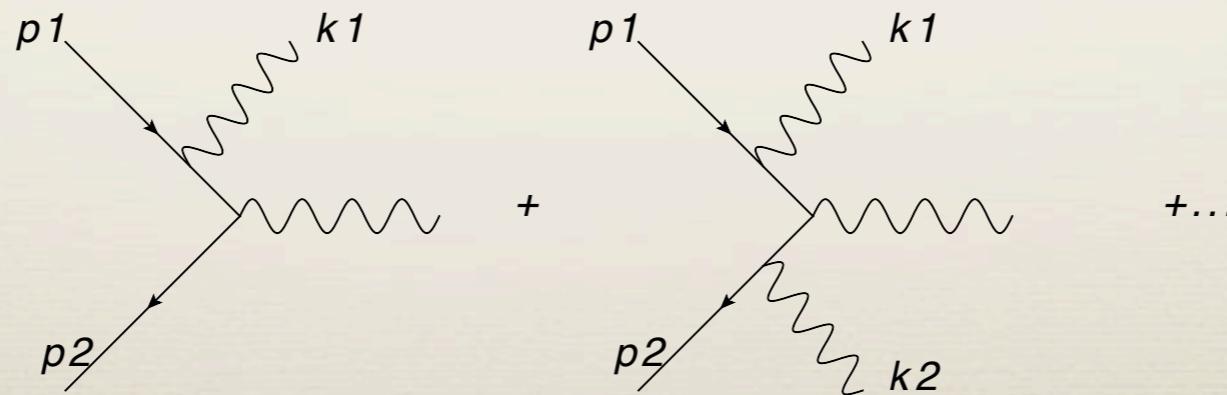
Low-p_T resummation

- * One more issue with result; go back and look at real radiation

$$\begin{aligned} p_T^2 &= \frac{\hat{t}\hat{u}}{\hat{s}} = \hat{s}(1-z)^2\lambda(1-\lambda) && \text{Fine if } p_T \text{ integrated, but what if} \\ \Rightarrow |\bar{\mathcal{M}}|^2 \times PS &\sim (p_T^2)^{-1-\epsilon} && \text{experiment selects low } p_T? \end{aligned}$$

$$\int_0^{p_T^{max}} (p_T^2)^{-1-\epsilon} \rightarrow \ln \frac{M_H}{p_T^{max}} \gg 1 \text{ for } M_H \gg p_T^{max}$$

- * Toy example of resummation for algebraic ease: $e^+e^- \rightarrow \gamma^*$ at leading-log level, then show full QCD result
- * Leading terms from multiple soft-photon radiation



Soft emissions in b-space

- * Both matrix elements and phase space simplify in this limit

Eikonal approximation for n-photon matrix-elements:

$$\mathcal{M}_n \propto g^n \mathcal{M}_0 \left\{ \frac{p_1 \cdot \epsilon_1 \dots p_1 \cdot \epsilon_n}{p_1 \cdot k_1 \dots p_1 \cdot k_n} + (-1)^n \frac{p_2 \cdot \epsilon_1 \dots p_2 \cdot \epsilon_n}{p_2 \cdot k_1 \dots p_2 \cdot k_n} \right\}$$

Phase-space for n-photon emission:

$$d\Pi_n \propto \nu(k_{T1}) d^2 k_{T1} \dots \nu(k_{Tn}) d^2 k_{Tn} \underbrace{\delta^{(2)} \left(\vec{p}_T - \sum_i \vec{k}_{Ti} \right)}_{\text{sum to Higgs } p_T}$$

$$\nu(k_T) = k_T^{-2\epsilon} \ln \left(\frac{s}{k_T^2} \right)$$

- * Would be independent if not for p_T constraint... Fourier transform accomplishes this

$$\int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{p}_T} \int d^2 k_{T1} f(k_{T1}) \dots d^2 k_{Tn} f(k_{Tn}) \delta^{(2)} \left(\vec{p}_T - \sum_i \vec{k}_{Ti} \right)$$

$$= \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{p}_T} [\tilde{f}(b)]^n, \quad \tilde{f}(b) = \int d^2 k_T e^{i\vec{b}\cdot\vec{k}_T} f(k_T)$$

Exponentiation

- * Product of matrix elements and phase space now exponentiates
Parisi, Petronzio NPB₁₅₄ (1979)

$$\frac{d\sigma}{d^2 p_T} = \sigma_0 \int \frac{d^2 b}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{p}_T} \tilde{\sigma}(b)$$

$$\tilde{\sigma}(b) = \exp \left\{ \frac{g^2}{4\pi^2} \int d^2 k_T e^{i\vec{b}\cdot\vec{k}_T} \left[\frac{\ln(s/k_T^2)}{k_T^2} \right]_+ \right\}$$

- * Large $b \Leftrightarrow$ small p_T ; inverse transform keeping leading terms

$$\frac{d\sigma}{dp_T^2} = \frac{\alpha}{\pi} \sigma_0 \frac{1}{p_T^2} \ln \frac{s}{p_T^2} \exp \left\{ -\frac{\alpha}{2\pi} \ln^2 \frac{s}{p_T^2} \right\}$$

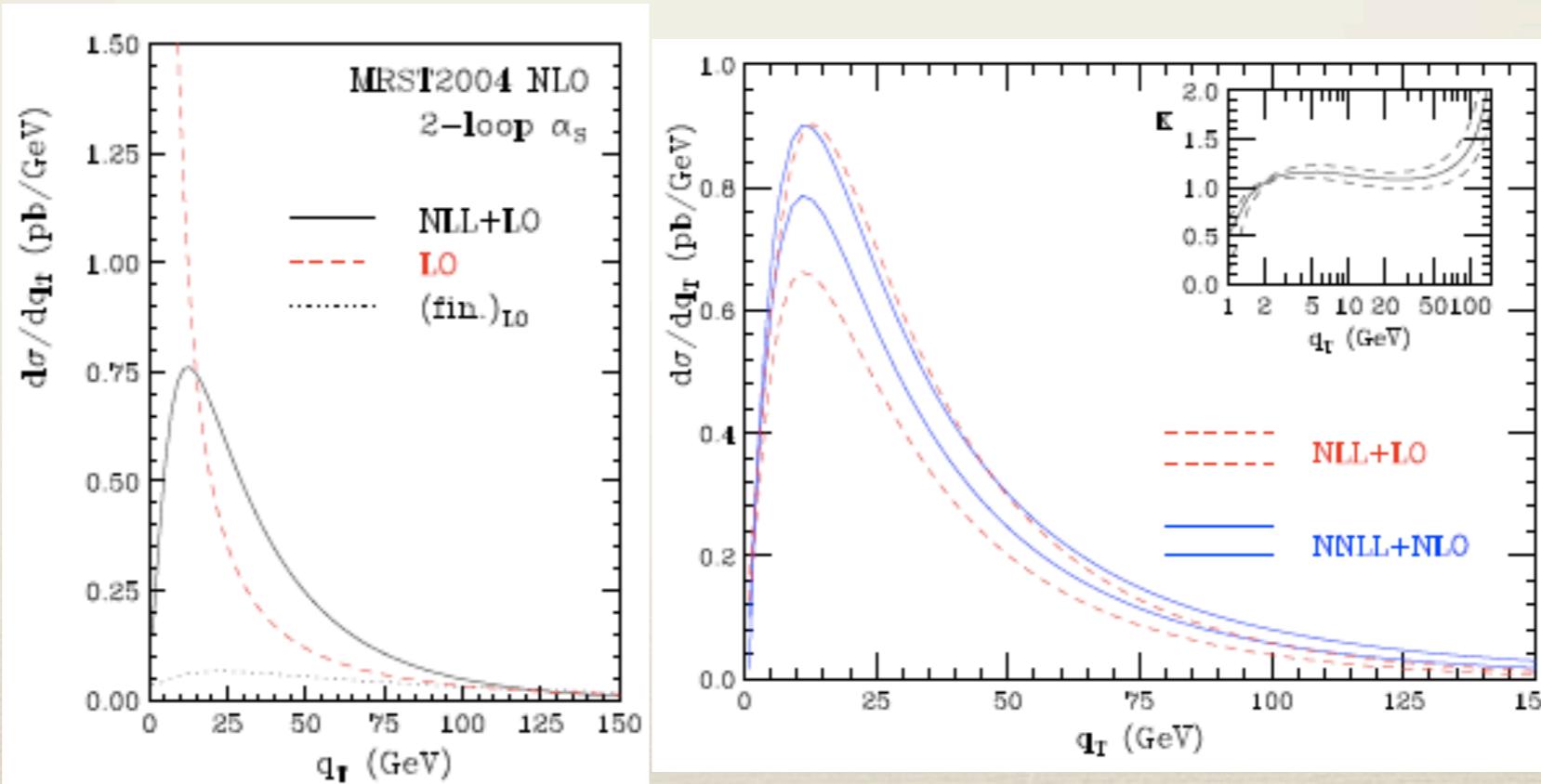


Low p_T in QCD

- * Can systematically improve to account for sub-leading logs, match to fixed-order at high p_T (Collins, Soper, Sterman '85; Berger, Qiu hep-ph/0210135; Bozzi et al. hep-ph/0302104, and others)

$$\frac{d\hat{\sigma}_{F_{ab}}^{(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) = \frac{M^2}{\hat{s}} \int \frac{d^2 b}{4\pi} e^{ib \cdot q_T} \mathcal{W}_{ab}^F(b, M, \hat{s}; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\begin{aligned} \mathcal{W}_N^F(b, M; \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2) &= \mathcal{H}_N^F(M, \alpha_s(\mu_R^2); M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \\ &\times \exp\{\mathcal{G}_N(\alpha_s(\mu_R^2), L; M^2/\mu_R^2, M^2/Q^2)\} . \end{aligned}$$



from Bozzi, Catani,
de Florian, Grazzini
hep-ph/0508068

Threshold improvement

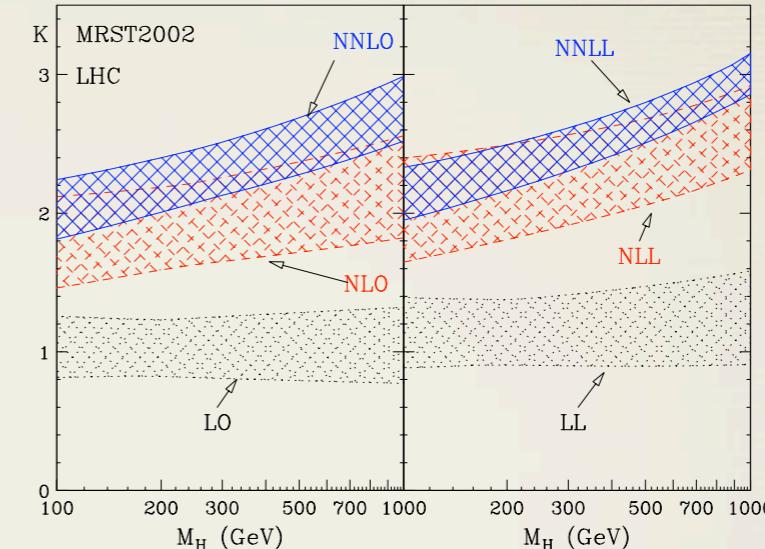
- * Resummation of \log, π^2 terms in inclusive result to all orders

(Sterman, '87; Catani, Trentadue '89; Magnea, Sterman '90)

Utilizes moment space: convolutions
of PDFs+cross section becomes product

$$\tilde{\sigma}(N) = \int_0^1 dz z^{N-1} \sigma(z) \Rightarrow \left[\frac{\ln^i(1-z)}{1-z} \right]_+ \rightarrow \ln^{i+1} N$$

$$\tilde{G}^{res}(N) \sim \exp \left\{ \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \times \left[2 \int_{\mu_F^2}^{M_H^2(1-z)^2} \frac{dq^2}{q^2} \underbrace{F(q^2, (1-z))}_{\text{calculable}} \right] \right\}$$



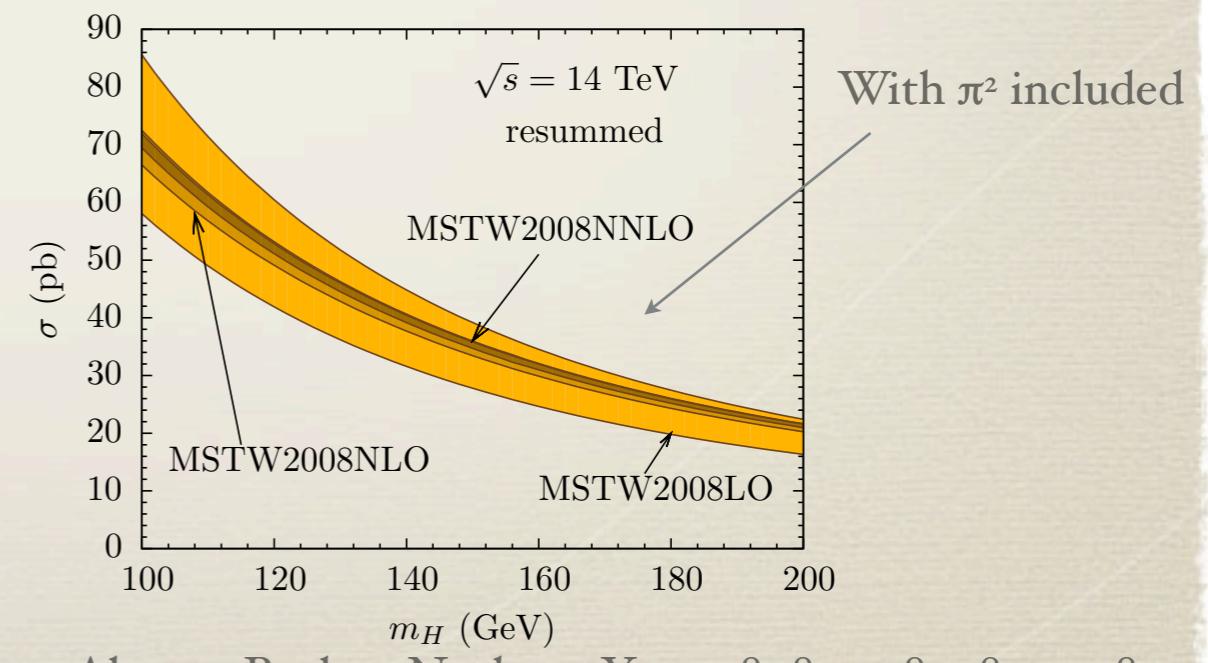
Catani, de Florian, Grazzini, Nason hep-ph/0306211

Utilize soft-collinear effective theory;

(C. Bauer, S. Fleming, D. Pirjol, I. Stewart
and others, 2000-2003; Becher, Neubert and others,
2006-2008)

	hard: $p_h \sim \sqrt{\hat{s}}(1, 1, 1)$, hard-collinear: $p_{hc} \sim \sqrt{\hat{s}}(\epsilon, 1, \sqrt{\epsilon})$, anti-hard-collinear: $p_{\bar{hc}} \sim \sqrt{\hat{s}}(1, \epsilon, \sqrt{\epsilon})$, soft: $p_s \sim \sqrt{\hat{s}}(\epsilon, \epsilon, \epsilon)$,
--	--

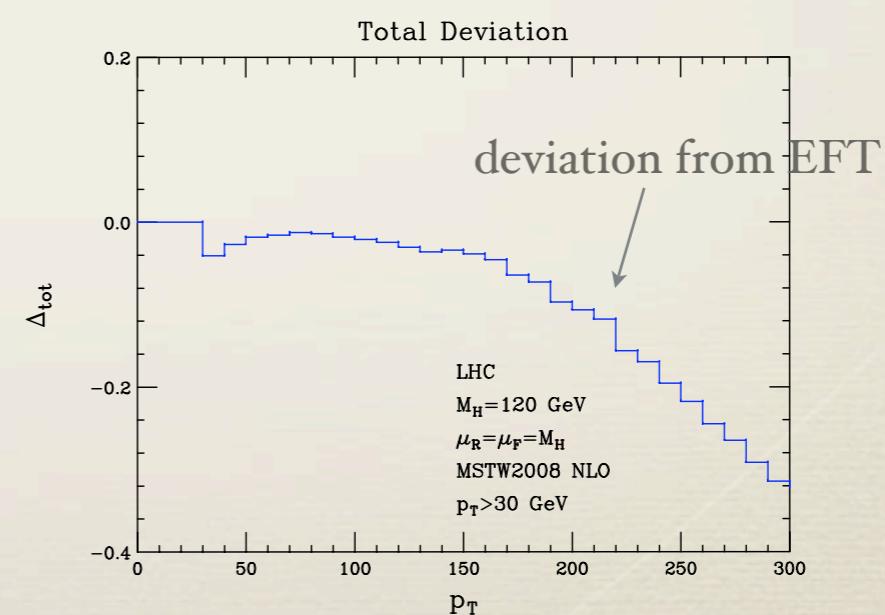
↔ Integrate out
hard modes
↔ Factorize collinear
modes into PDFs



Ahrens, Becher, Neubert, Yang 0808.3008, 0809.4283

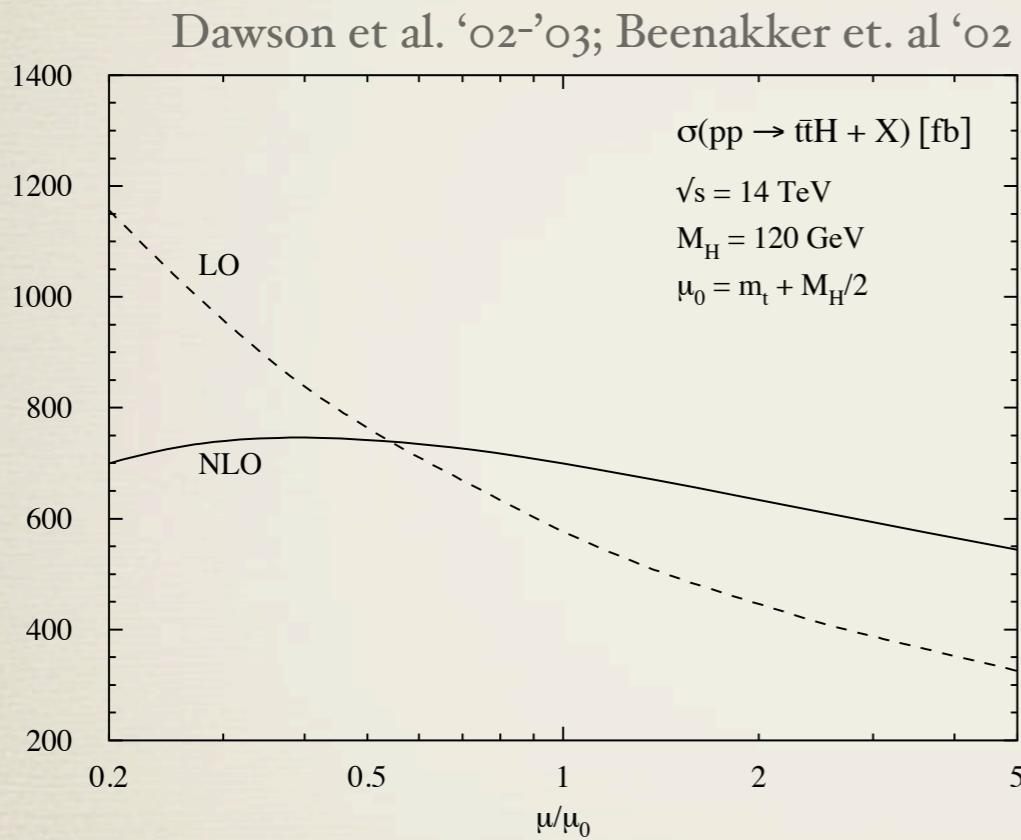
Summary of gg \rightarrow h

- * Remaining scale uncertainty in inclusive cross section: $\pm 10\%$ from fixed-order, $\pm 3\%$ from RG-improved calculation
- * Kinematic cuts strongly affect both central value and scale uncertainty, must be accounted for carefully
- * EFT arising from integrating top-quark a good framework in which to study QCD effects; valid to $< 1\%$ up to $M_H=2m_t$, but beware cuts...
- * 2-4 larger than LO prediction; control of QCD, EW vital! (more on EW later...)
- * PDF errors discussed in experimental analysis section

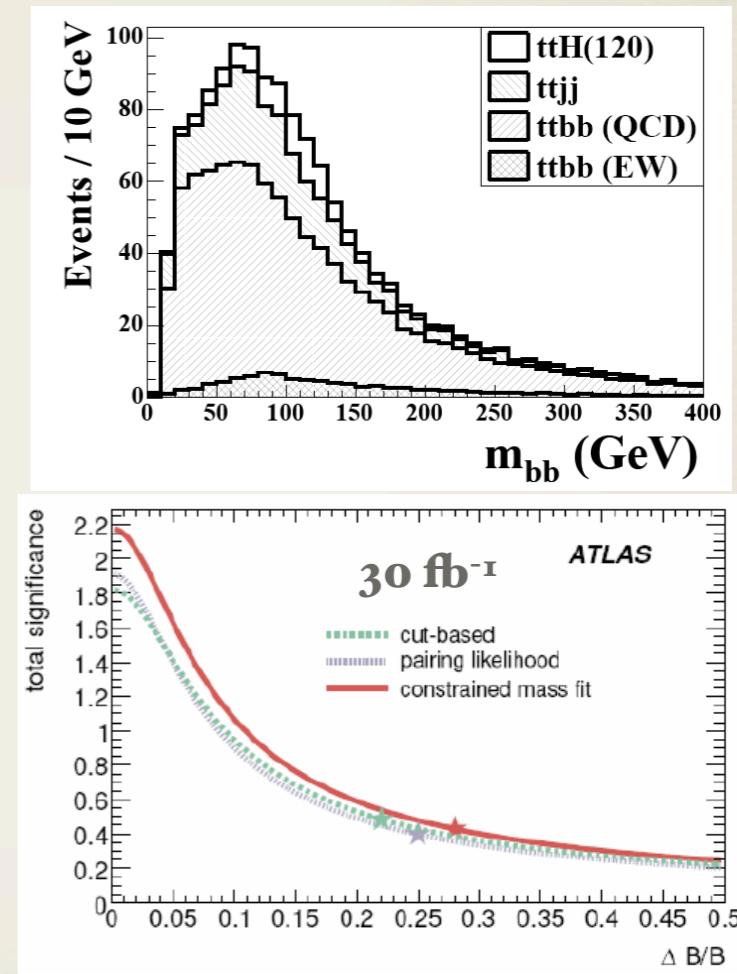


tth associated production

- * Allows measurement of htt Yukawa coupling, and also hbb coupling through $h \rightarrow bb$ decay



NLO corrections reduce scale dependence

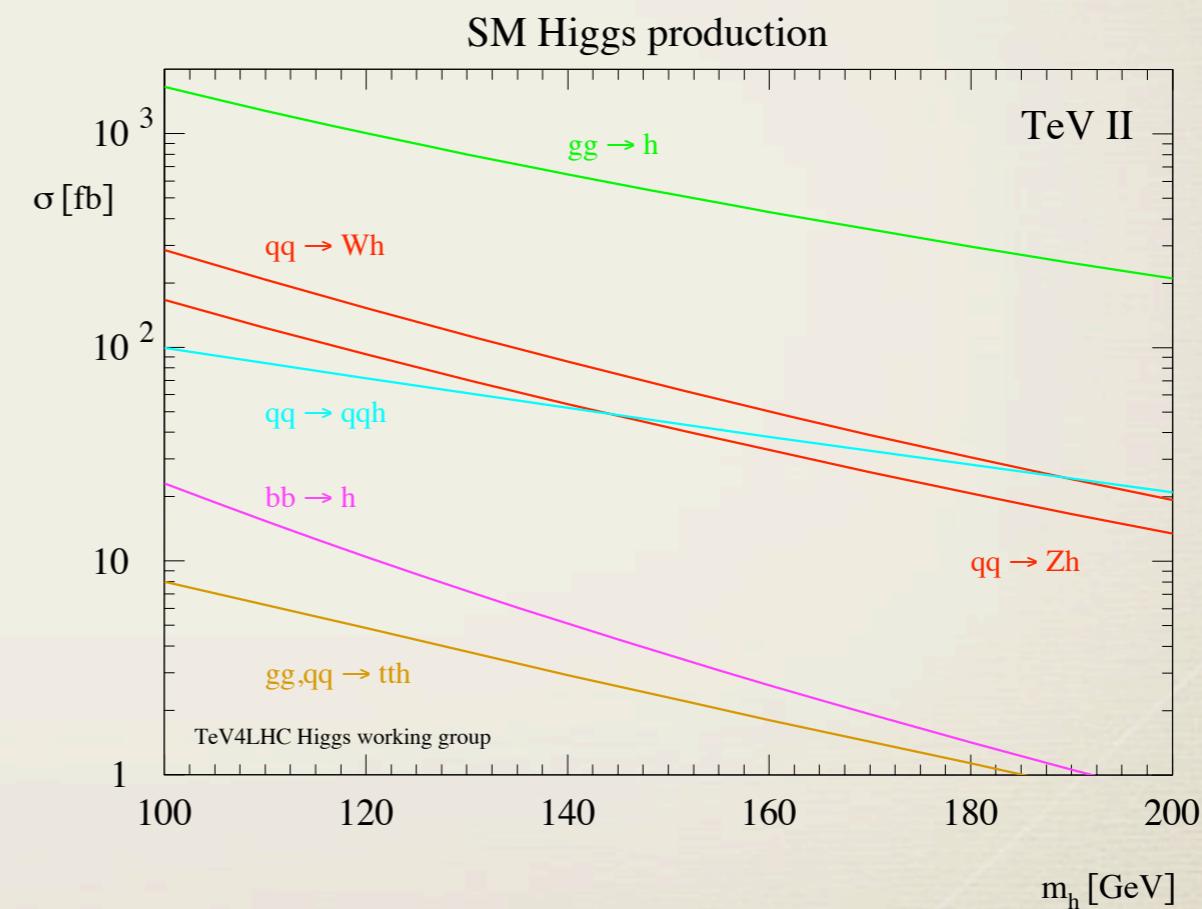
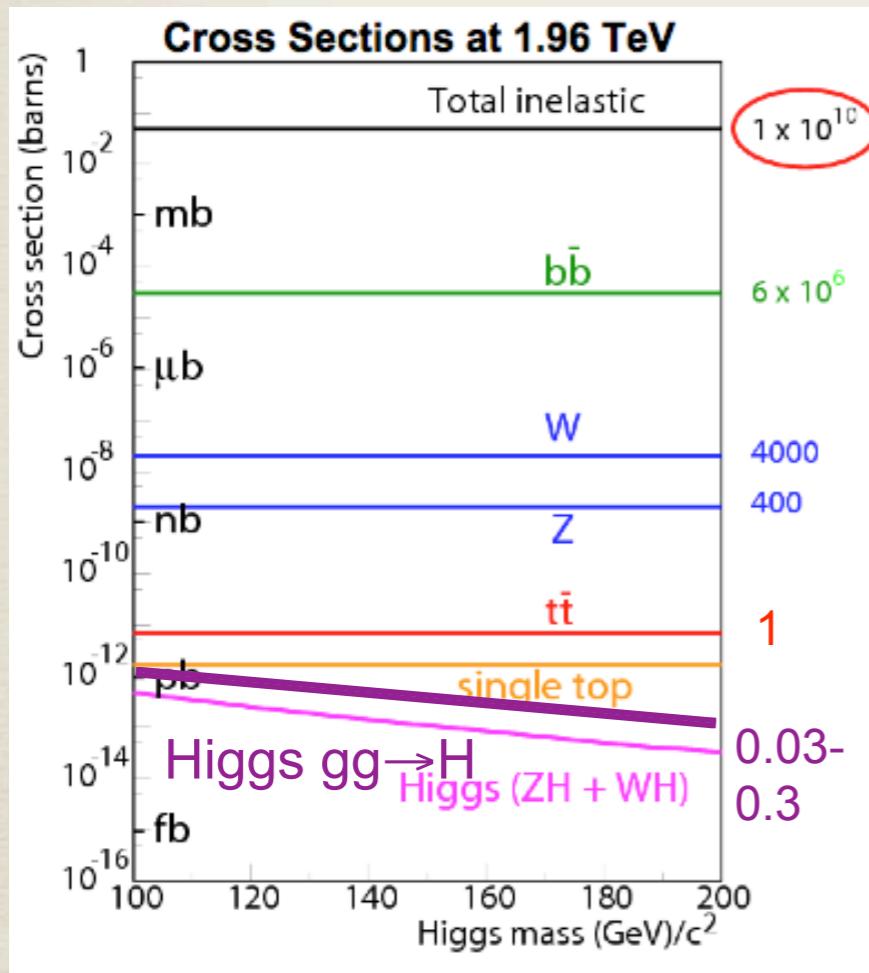


Recent re-analysis discouraging;
large SM ttbb background; high
luminosity only

Searches at the Tevatron and LHC

Tevatron analysis overview

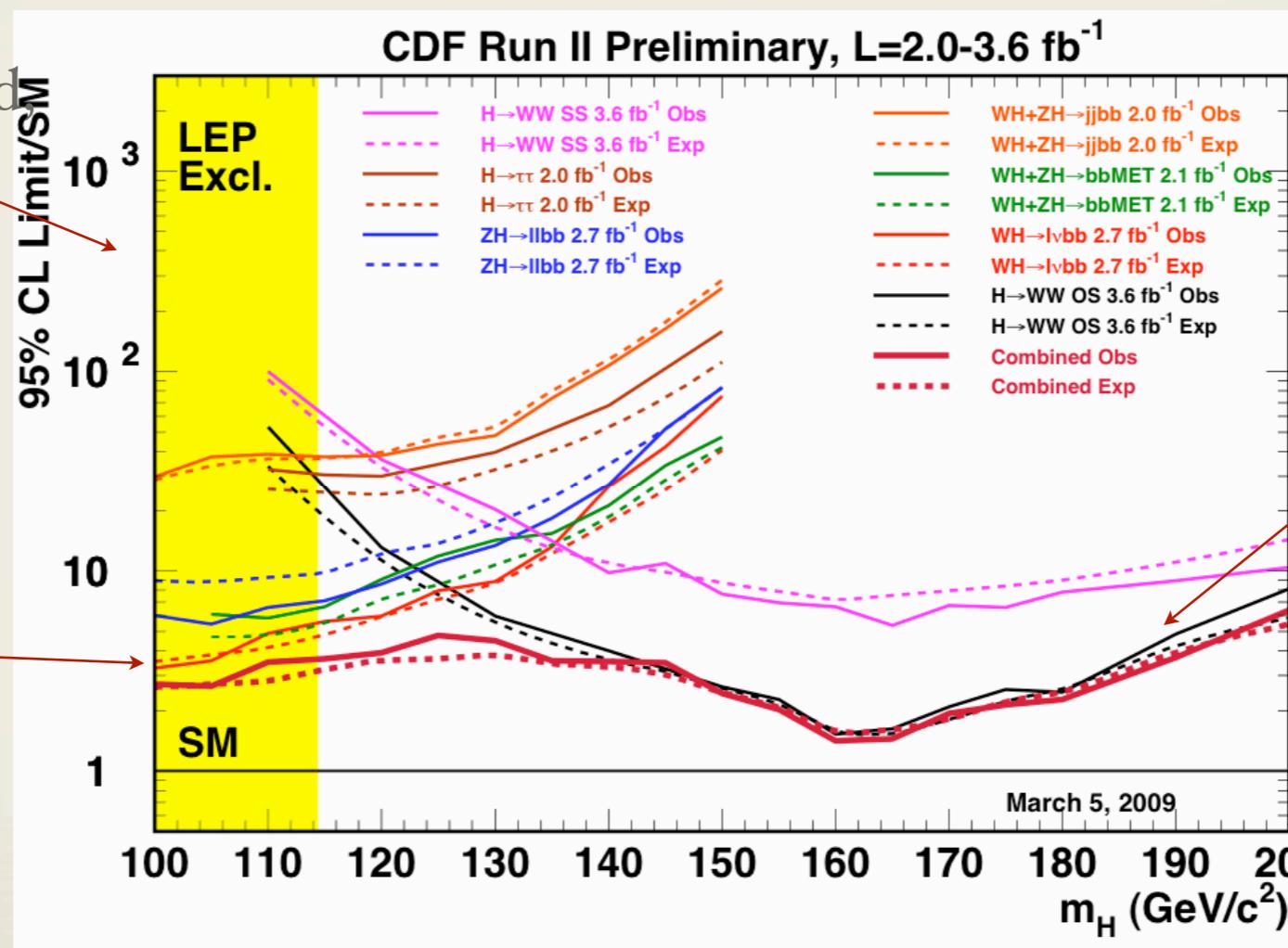
- * Inclusive $gg \rightarrow h \rightarrow bb$ not feasible at low masses
- * WBF only slightly adds to analyses designed for other channels



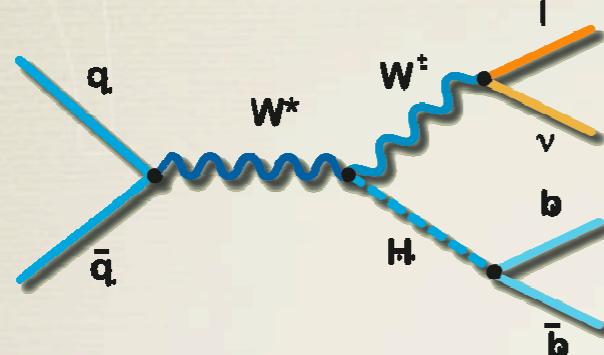
Combined exclusion limit

- * No observation, so collaborations report 95% C.L. exclusion by combining many possible channels

Cross section excluded
normalized to SM



Wh \rightarrow lvbb analysis



Basic acceptance cuts :

- $p_T^l > 20$ GeV
- $E_T > 20$ GeV
- 2-3 jets, 1-2 b-tags
- $p_T^j > 20$ GeV

Process	1 tag	2 tags
All Pretag Cands.	50644.0 ± 0.0	57174.0 ± 0.0
WW	56.2 ± 6.2	0.4 ± 0.1
WZ	23.0 ± 1.7	4.8 ± 0.5
ZZ	0.8 ± 0.1	0.2 ± 0.0
TopLJ	121.3 ± 17.1	23.8 ± 3.9
TopDil	48.8 ± 6.8	14.1 ± 2.3
STopT	64.0 ± 9.3	1.8 ± 0.3
STopS	40.6 ± 5.7	12.8 ± 2.1
Z+jets	37.4 ± 5.5	2.1 ± 0.3
Total MC	392.0 ± 35.0	59.9 ± 7.5
Wbb	538.7 ± 162.5	70.3 ± 22.5
Wcc/Wc	489.1 ± 150.9	6.8 ± 2.3
Total HF	1027.8 ± 312.3	77.1 ± 24.7
Mistags	458.0 ± 57.9	2.2 ± 0.6
Non-W	135.5 ± 54.2	9.0 ± 3.6
Total Prediction	2013.3 ± 324.1	148.2 ± 26.1
WH100	9.5 ± 0.8	2.9 ± 0.3
WH105	8.6 ± 0.7	2.7 ± 0.3
WH110	7.6 ± 0.6	2.4 ± 0.3
WH115	6.3 ± 0.5	2.0 ± 0.2
WH120	4.9 ± 0.4	1.6 ± 0.2
WH125	4.0 ± 0.3	1.3 ± 0.2
WH130	3.1 ± 0.3	1.0 ± 0.1
WH135	2.3 ± 0.2	0.7 ± 0.1
WH140	1.5 ± 0.1	0.5 ± 0.1
WH145	1.0 ± 0.1	0.3 ± 0.0
WH150	0.7 ± 0.1	0.2 ± 0.0
Observed	1998.0 ± 0.0	156.0 ± 0.0

From CDF, after event selection

``Estimating the background contribution after applying the event selection to the WH candidate sample is an elaborate process''

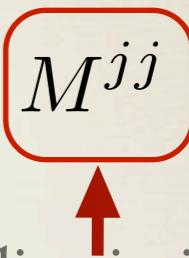
W+jets: normalization from data; heavy-flavor fraction from ALPGEN for shape (tree-level)+data for norm.; Do also uses NLO to check

Combined theory
+experiment error

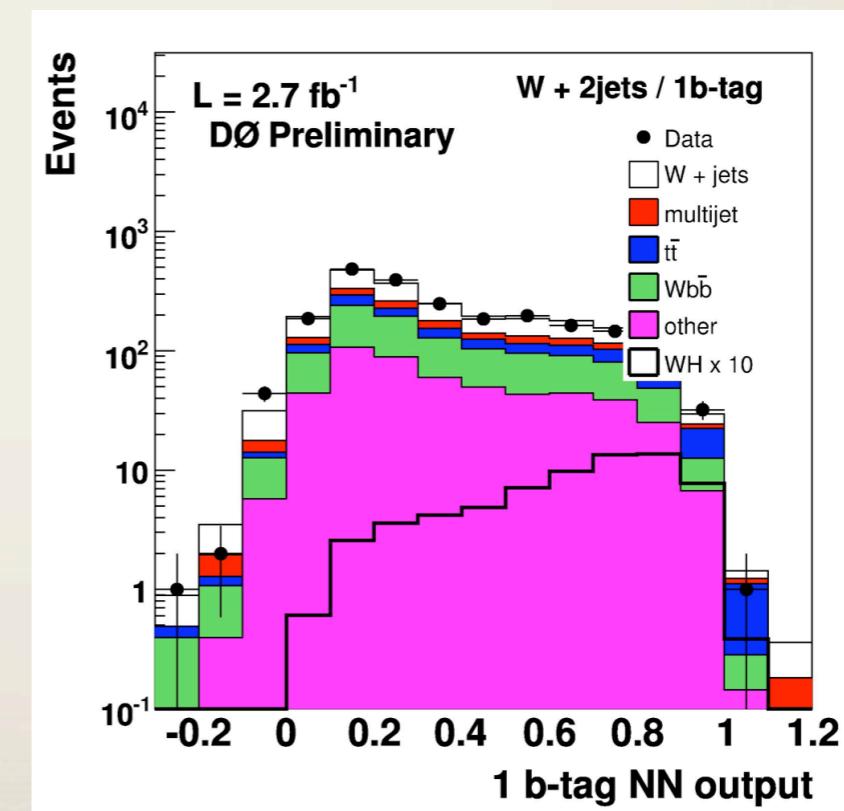
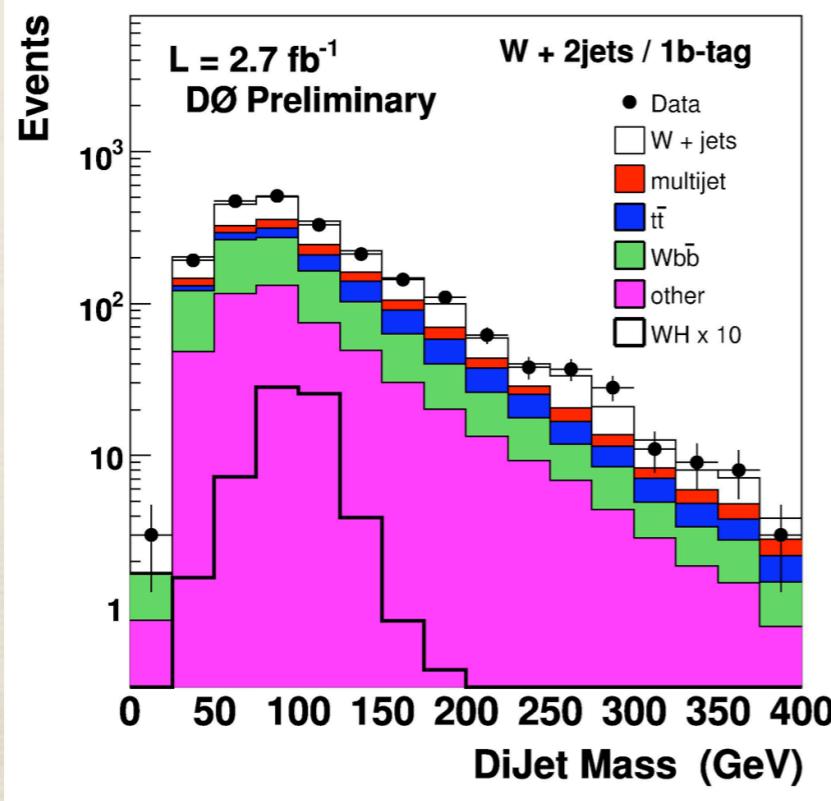
Kinematic discriminants

- * Use neural networks or “boosted decision trees” that search for optimal combination of kinematic variables

D0 input variables: $p_T^{j1}, p_T^{j2}, \Delta R^{jj}, \Delta\phi^{jj}, p_T^{jj}, p_T^{l+E_T}, M^{jj}$



Do “most-discriminating”

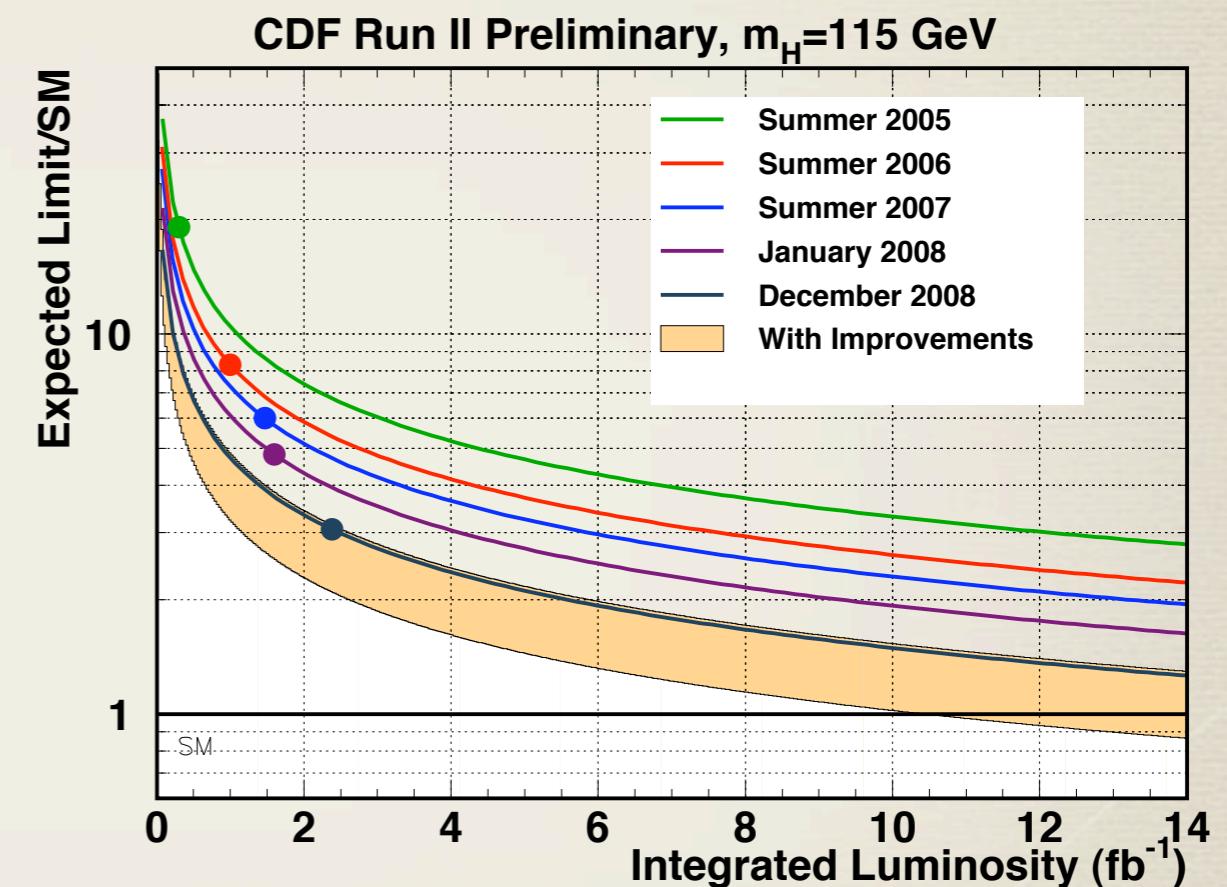
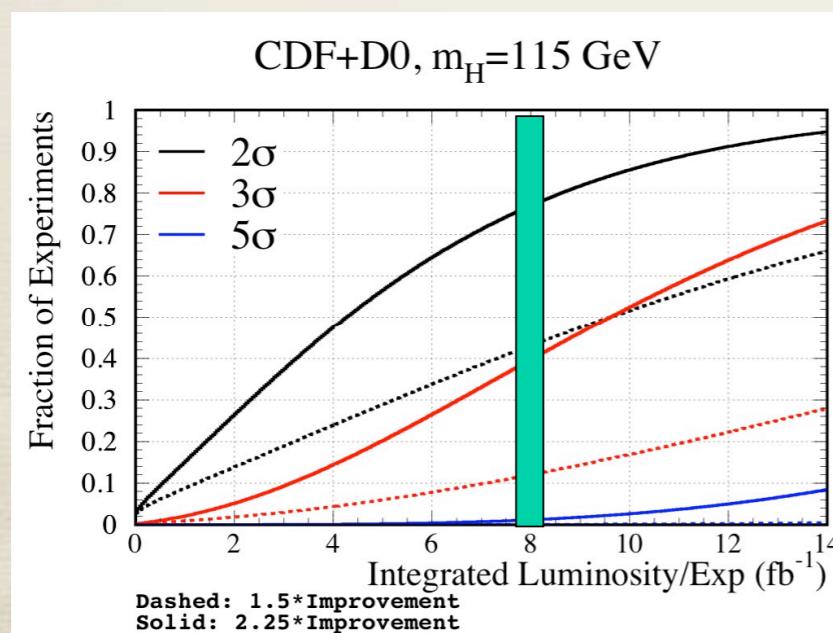


Low-mass limits and projections

- * Analysis improvements expected: better dijet mass resolution, increased acceptance

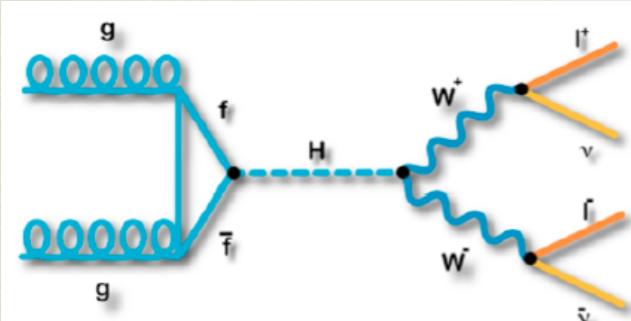
Results at $m_H = 115\text{GeV}$: 95%CL Limits/SM				
Analysis	Lum (fb^{-1})	Higgs Events	Exp. Limit	Obs. Limit
CDF NN+ME+BDT	2.7	8.4	4.8	5.8
DØ ME+NN new	2.7	13.3	6.7	6.4

from M. Herndon, LoopFest 2009



Likely to have exclusion, possibility of 3σ at low mass (depends on Nature...)

$h \rightarrow WW \rightarrow l\nu l\nu$



Basic acceptance cuts (CDF) :

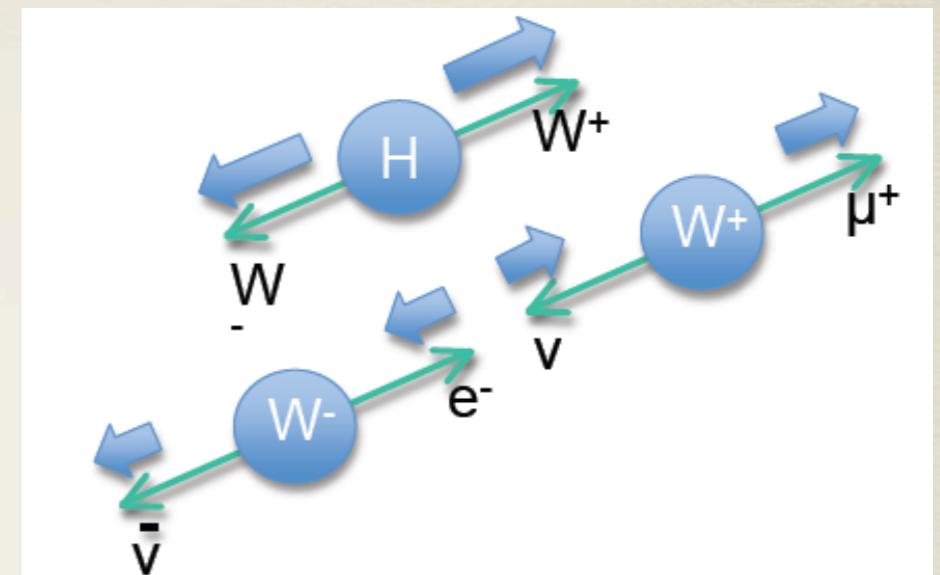
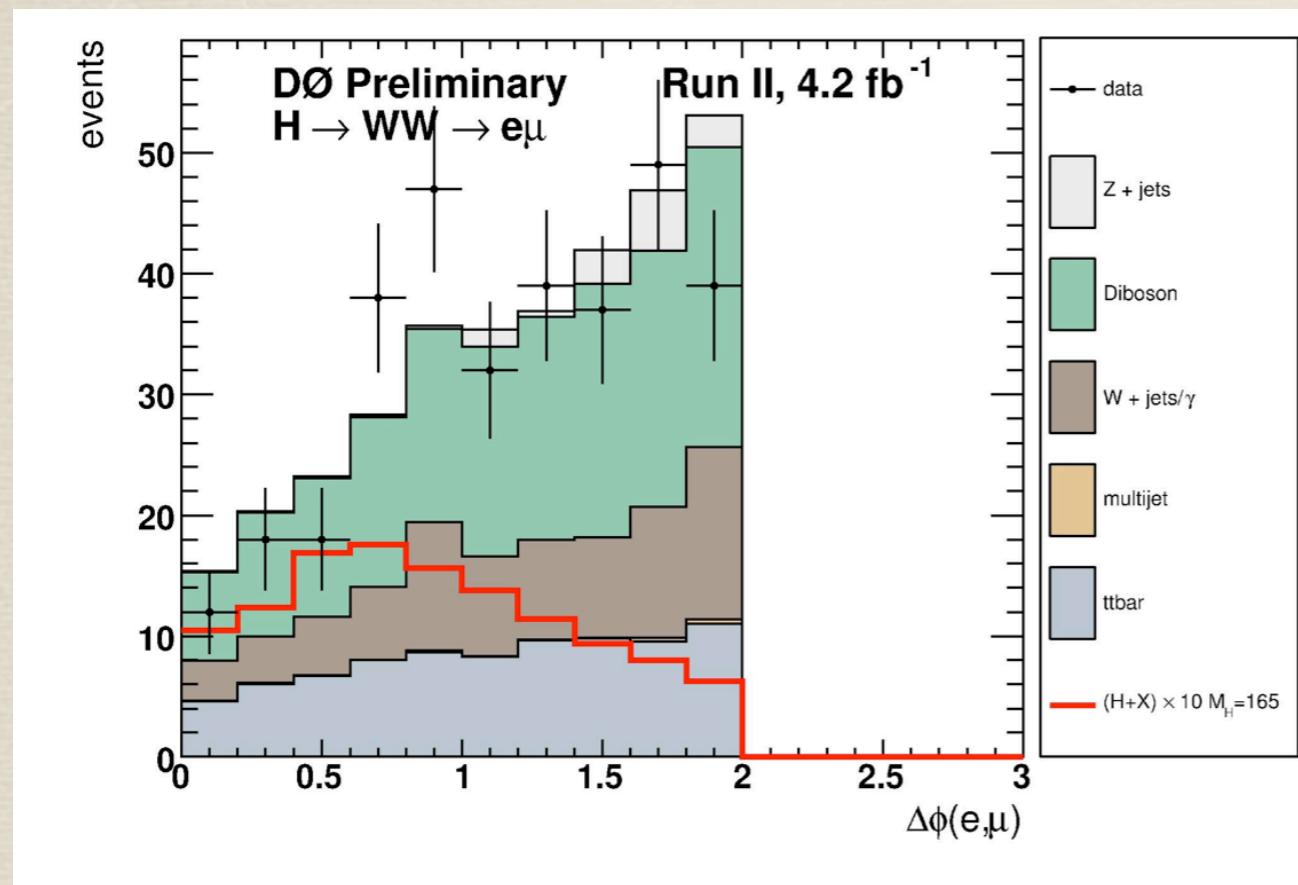
- $p_T^{l1} > 20$ GeV
- $p_T^{l2} > 10$ GeV
- $E_T > 15 - 25$ GeV
(for various final states)
- Look separately at 0,1,2+ jet bins

	ee pre-selection	ee final	$e\mu$ pre-selection	$e\mu$ final
$Z \rightarrow ee$	218695 ± 704	108 ± 14	280.6 ± 3.3	$0.0^{+0.1}_{-0.0}$
$Z \rightarrow \mu\mu$	—	—	274.6 ± 0.9	5.8 ± 0.1
$Z \rightarrow \tau\tau$	1135 ± 16	1.4 ± 0.5	3260 ± 3	7.3 ± 0.1
$t\bar{t}$	131.4 ± 1.4	39.9 ± 0.8	272.0 ± 0.3	82.5 ± 0.2
<u>W+jets</u>	241 ± 5	98 ± 3	183 ± 4	78.6 ± 2.8
<u>WW</u>	172.2 ± 2.6	66.8 ± 1.6	421.2 ± 0.1	154.7 ± 0.1
WZ	112.5 ± 0.2	9.68 ± 0.05	20.5 ± 0.1	6.6 ± 0.1
ZZ	98.2 ± 0.2	7.68 ± 0.07	5.3 ± 0.1	0.60 ± 0.01
Multijet	1351 ± 55	$1.7^{+2.0}_{-1.7}$	279 ± 168	$1.1^{+9.6}_{-1.1}$
Signal ($M_H = 165$ GeV)	9.45 ± 0.01	6.13 ± 0.01	17.1 ± 0.01	12.2 ± 0.1
Total Background	221937 ± 707	332 ± 15	4995 ± 168	337 ± 10
Data	221530	336	4995	329

from Do

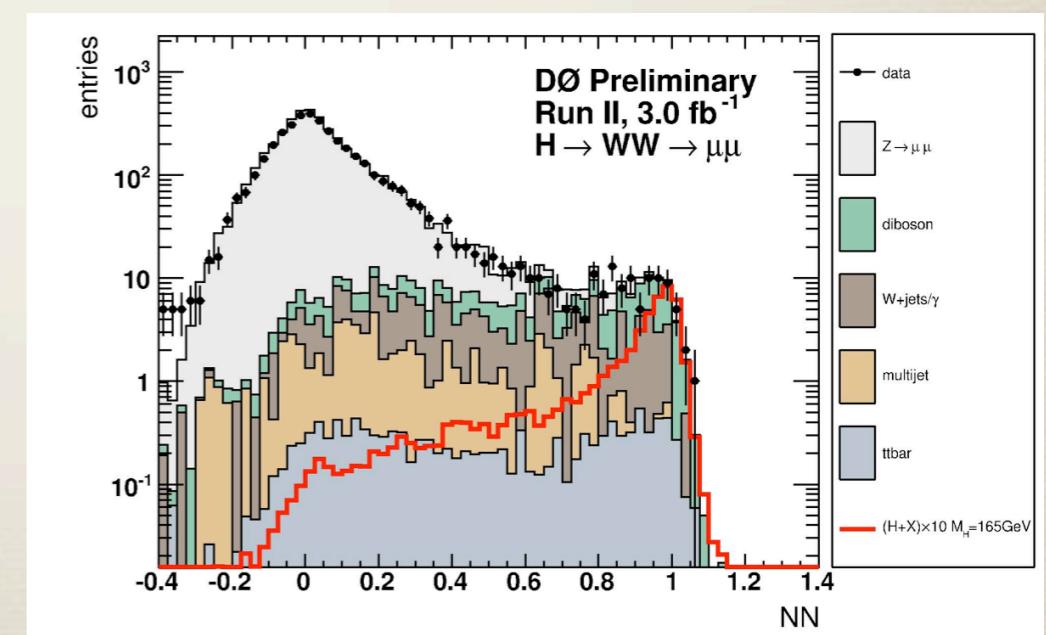
tt: affects 2-jet bin; taken from NLO calculations
W+jets: jet fakes lepton; from data-driven methods
WW: taken from NLO calculation

Kinematic discriminants



A primary handle for o-jet bin: $\Delta\varphi_{ll}$
Spin correlation: leptons in same direction

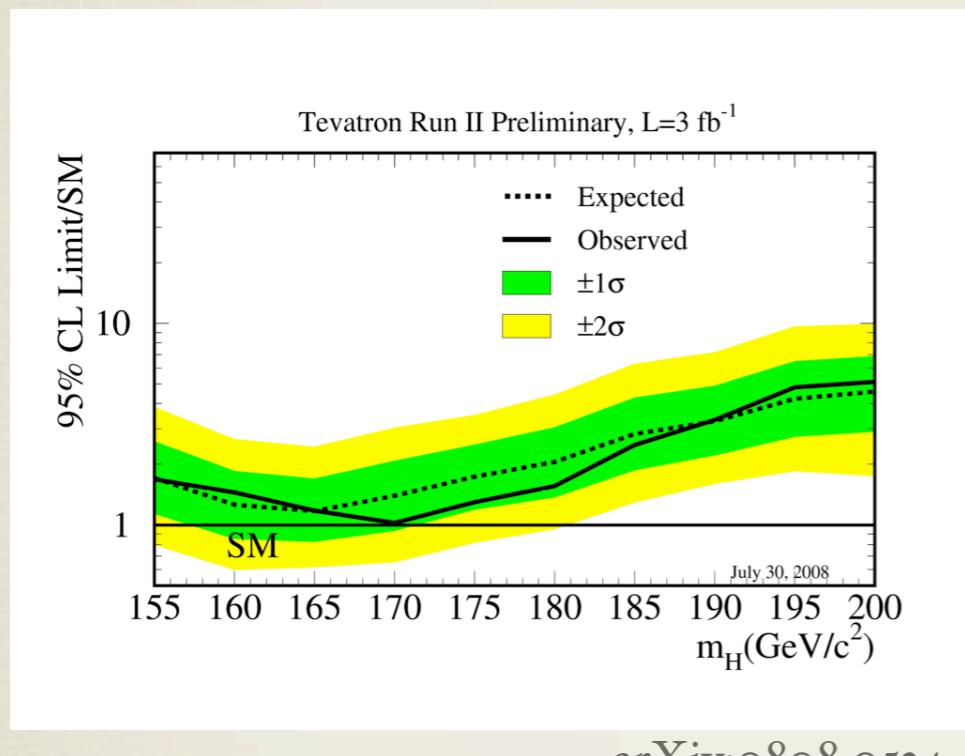
NN Analysis Variables	
p_T of leading lepton	$p_T(\ell_1)$
p_T of trailing lepton	$p_T(\ell_2)$
Minimum of both lepton qualities	$\min(q_{\ell 1}, q_{\ell 2})$
Vector sum of the transverse momenta of the leptons:	$p_T(\ell_1) + p_T(\ell_2)$
Scalar sum of the transverse momenta of the jets:	$H_T = \sum_i p_T(\text{jet}_i) $
Invariant mass of both leptons	$M_{\text{inv}}(\ell_1, \ell_2)$
Minimal transverse mass of one lepton and E_T	M_T^{\min}
Missing transverse energy	E_T
Scalar transverse energy	E_T^{scalar}
Azimuthal angle between selected leptons	$\Delta\phi(\ell_1, \ell_2)$
Solid angle between selected leptons ($e\mu$ only)	$\Delta\Theta(\ell_1, \ell_2)$
ΔR between selected leptons ($e\mu$ only)	$\Delta R(\ell_1, \ell_2)$
Azimuthal angle between leading lepton and E_T	$\Delta\phi(E_T, \ell_1)$
Azimuthal angle between trailing lepton and E_T	$\Delta\phi(E_T, \ell_2)$



What is being excluded?

Pre-2008 studies: no EW corrections included in SM Higgs prediction

Combined CDF, Do results (2008)



First limits: $M_H=170 \text{ GeV}$ excluded

What went into the SM prediction:

- Same K-factors assumed for top, EW
⇒ checked by 3-loop mixed QCD-EW described previously
- Same QCD corrections for t,b
⇒ $K_{tb} \sim 1.5$, $K_{tt} > 3$; needed fixing
- Old PDFs (MRST 2002)

PDF changes: new data sets, different α_s , heavy-quark masses

original	MRST 2006 PDFs	K_{tb}, K_{bb}	EW effects
0.3542	0.3650	0.3868	0.3943

What they used →

← Update #1

Fun with PDFs

MSTW 2008 PDF release arXiv:0901.0002

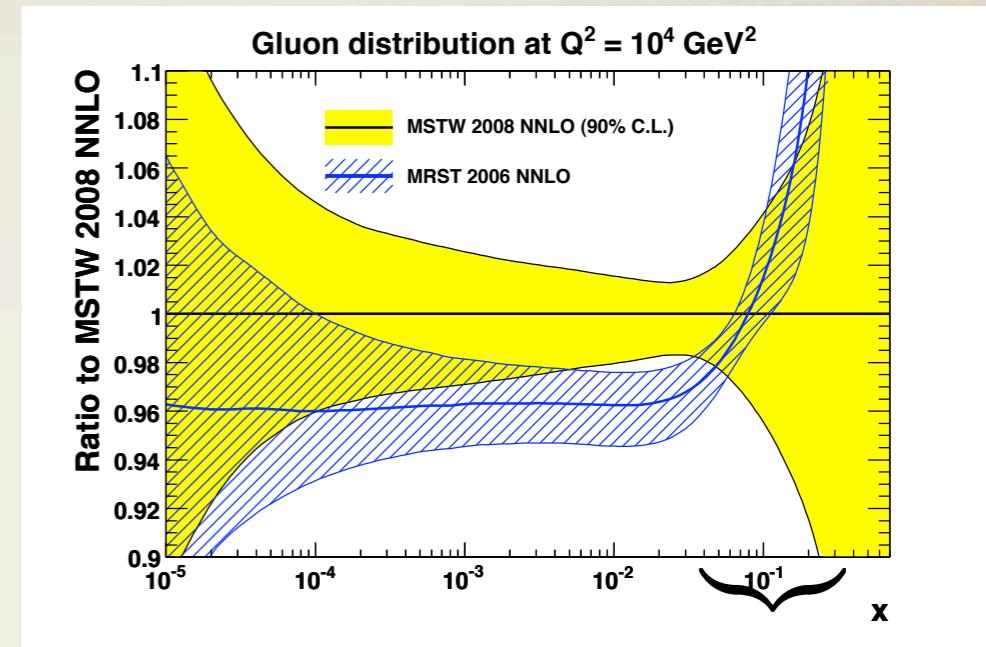
- Run II inclusive jet data
- Decrease of $\alpha_s(M_Z)$ from 0.119 → 0.117
- Gluon density decreased at $x \sim 0.1$
- gg luminosity error increased from 5% → 10%

$M_H=170$ GeV:

MRST 2001	MRST 2004	MRST 2006	MSTW 2008
0.3833	0.3988	0.3943	0.3444

~10-15% decrease in predicted cross section !

$$\sigma_{gg \rightarrow h} \sim \alpha_s^{2-3} \times f_g^2 \Rightarrow \text{very sensitive to these changes!}$$

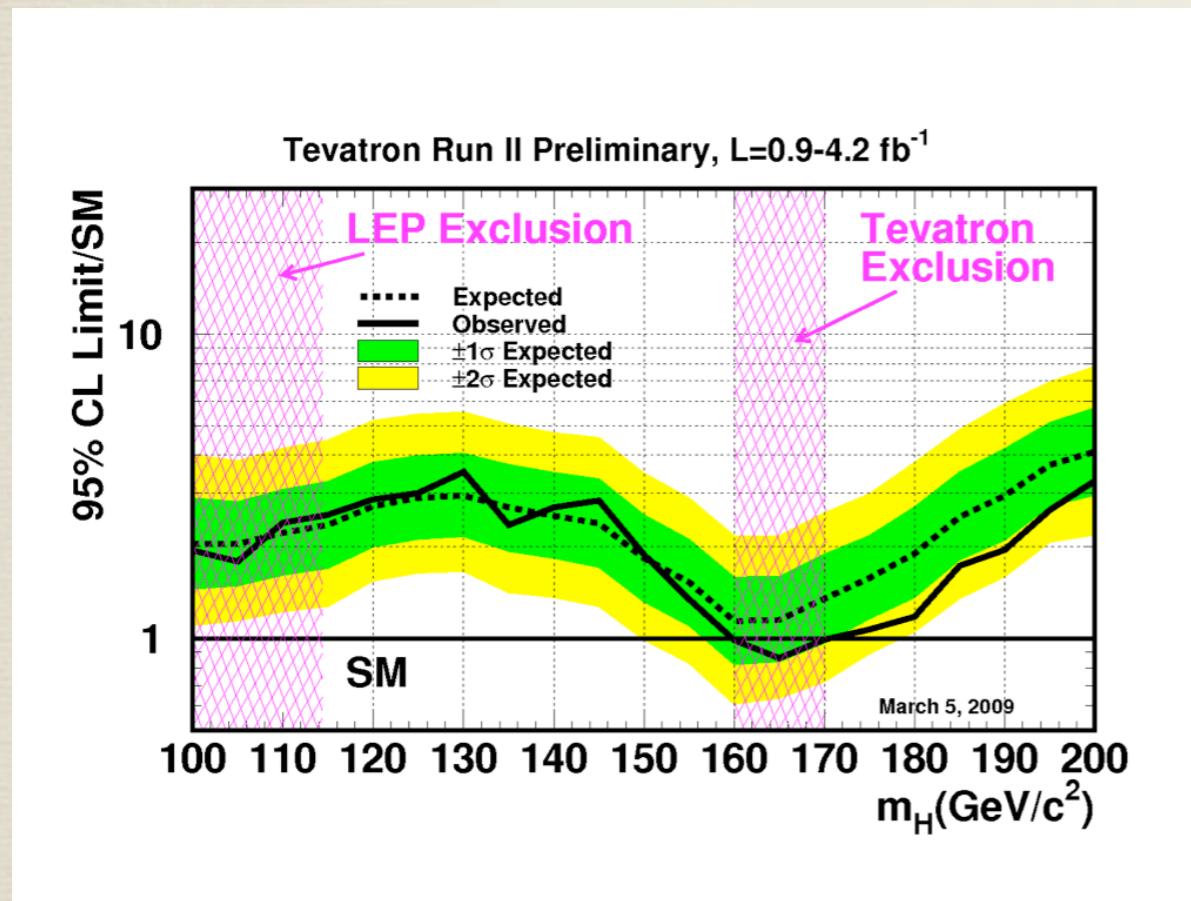


Changes in relevant region for Tevatron analyses from Tevatron jet data

PDFs+errors a vital aspect of any experimental analysis or theoretical prediction!

High-mass exclusion

- * Combine CDF+Do exclusion limits: $160 \leq M_H \leq 170$ GeV at 95% CL



95%CL Limits/SM					
$M_{\text{Higgs}}(\text{GeV})$	155	160	165	170	175
Method 1: Exp	1.5	1.1	1.1	1.4	1.6
Method 1: Obs	1.4	0.99	0.86	0.99	1.1
Method 2: Exp	1.5	1.1	1.1	1.3	1.6
Method 2: Obs	1.3	0.95	0.81	0.92	1.1

Solidly exclude SM at 165 GeV

0 Jet Uncertainties	$gg \rightarrow H$
Cross Section	
Scale	10.9%
PDF Model	5.1% (circled)
Total	12.0%

Factor of 2 larger,
not yet updated

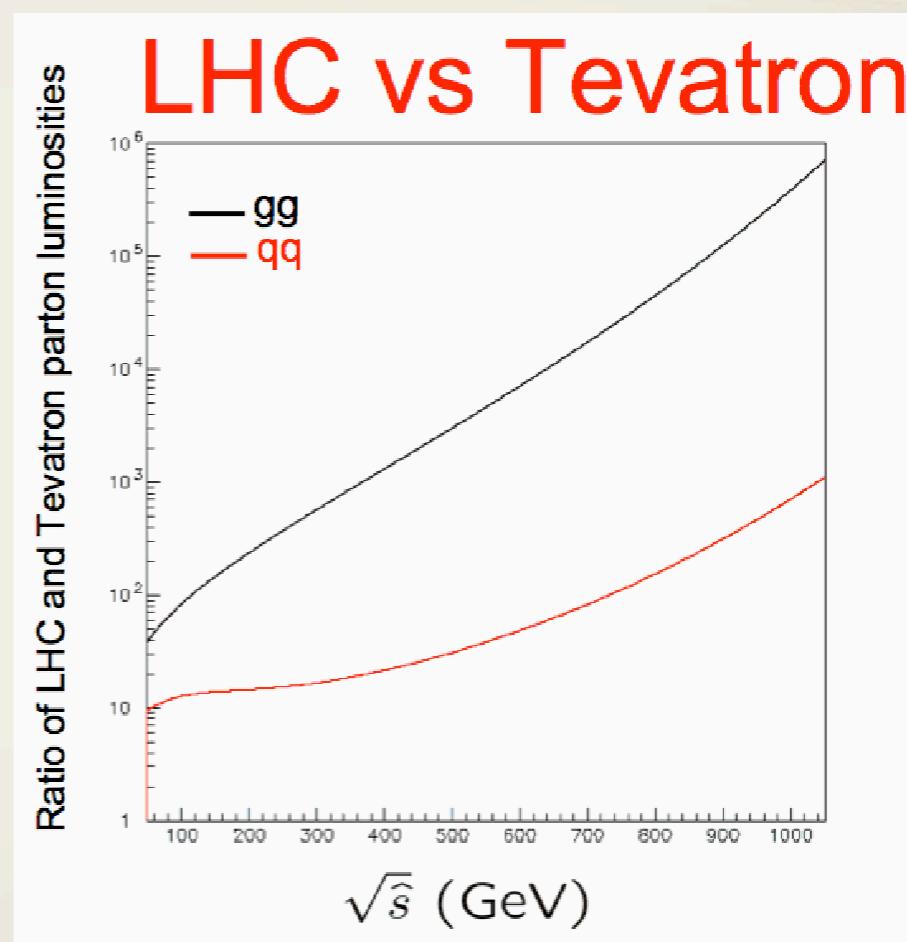
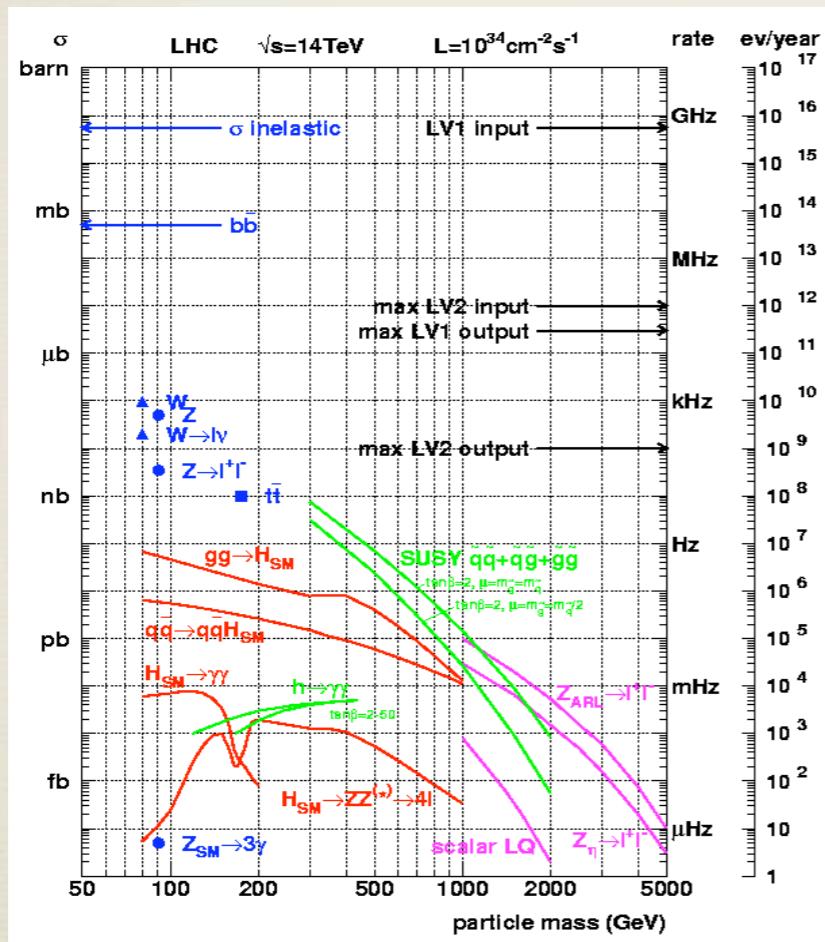
Must carefully look at results and talk
to experimental colleagues to
understand what goes into analysis!

LHC physics overview

- * Qualitative change; gluons now overwhelm scattering rate

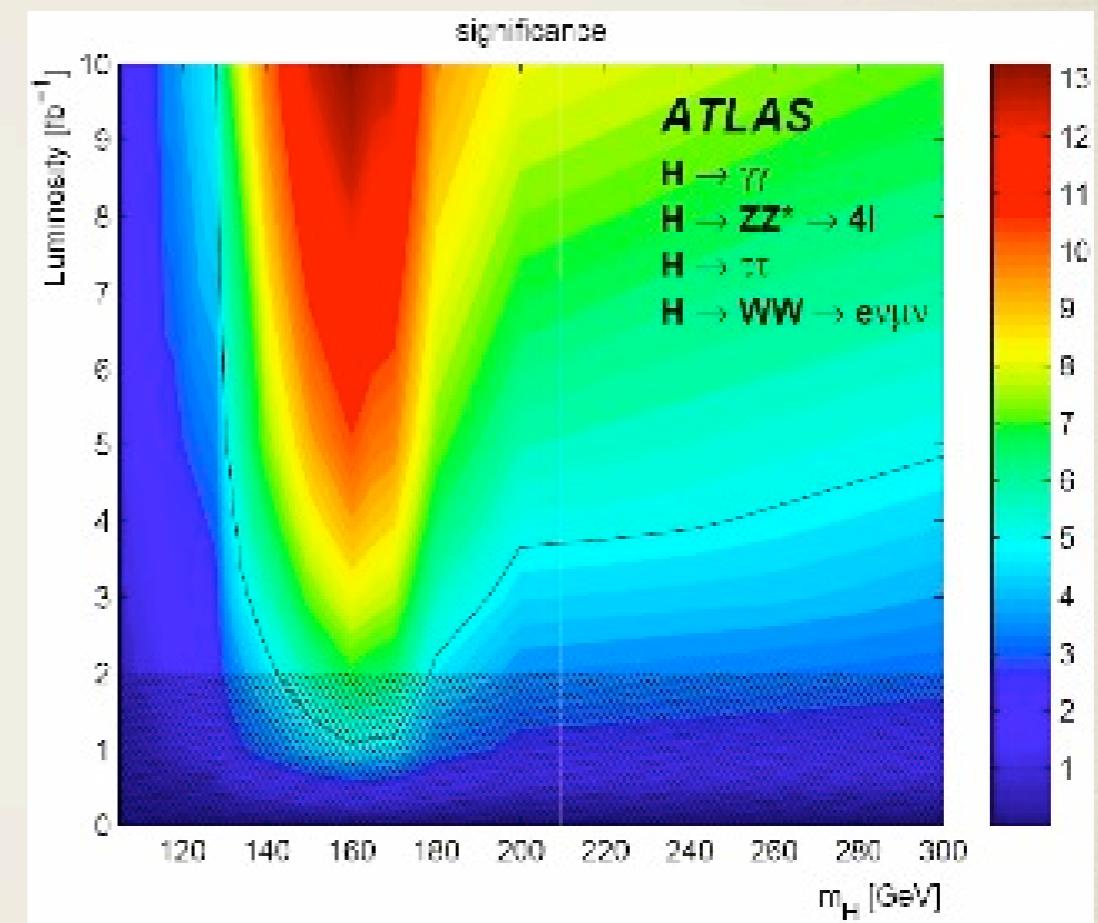
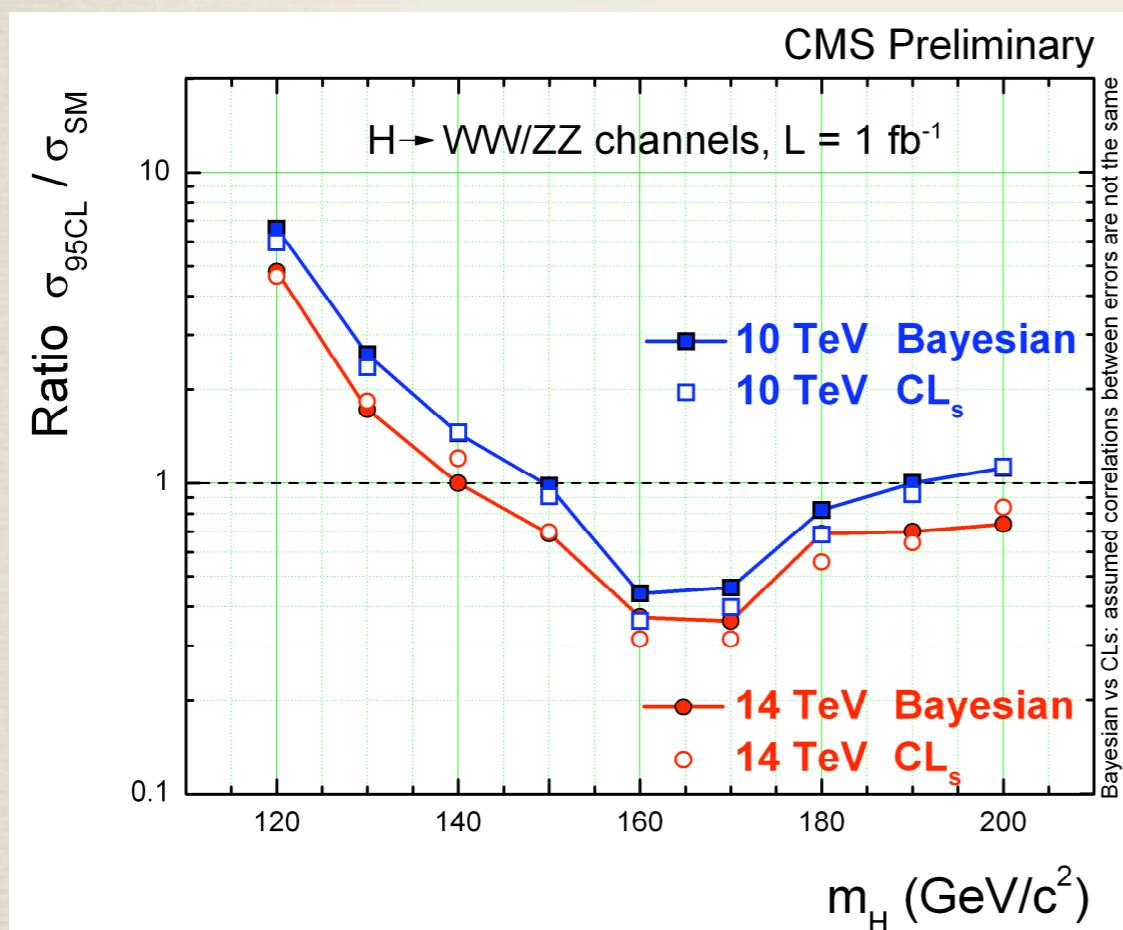
Background

Higgs



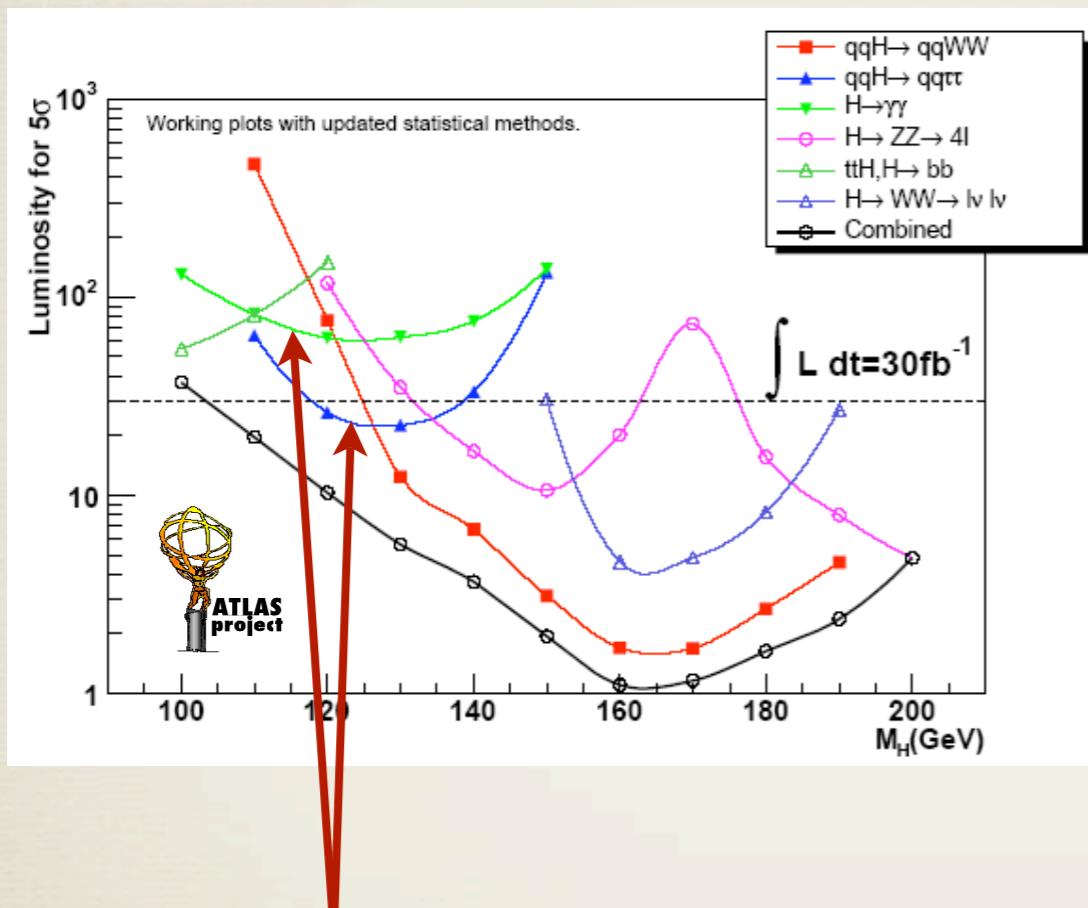
LHC summary: low fb^{-1}

- * Will reproduce expected Tevatron exclusion with 1 fb^{-1}



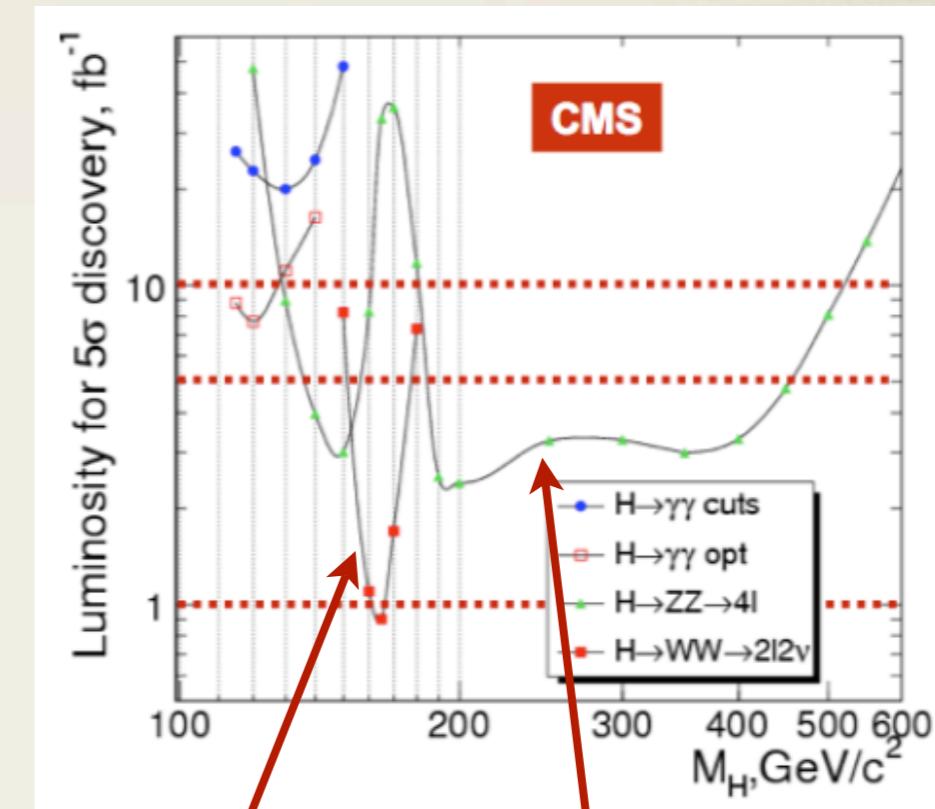
LHC summary: high fb^{-1}

- * Entire mass range covered, much with multiple modes



$h \rightarrow \gamma\gamma$, WBF $h \rightarrow \tau\tau$ cover low mass range

$M_H > 200 \text{ GeV}$: only few fb^{-1}
 $M_H < 120 \text{ GeV}$: $20-30 \text{ fb}^{-1}$

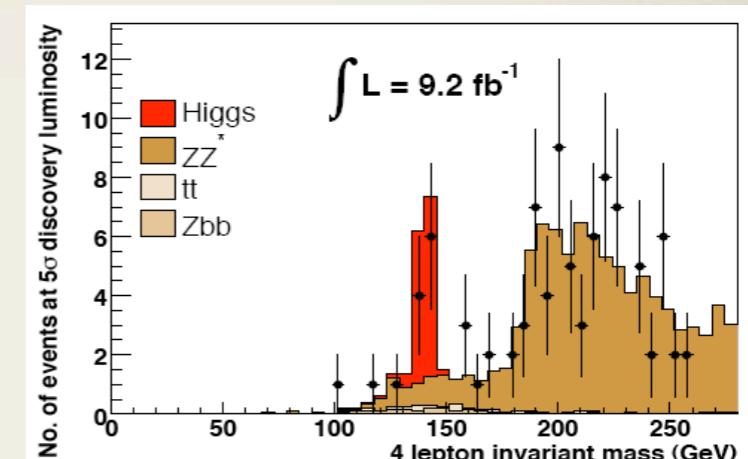
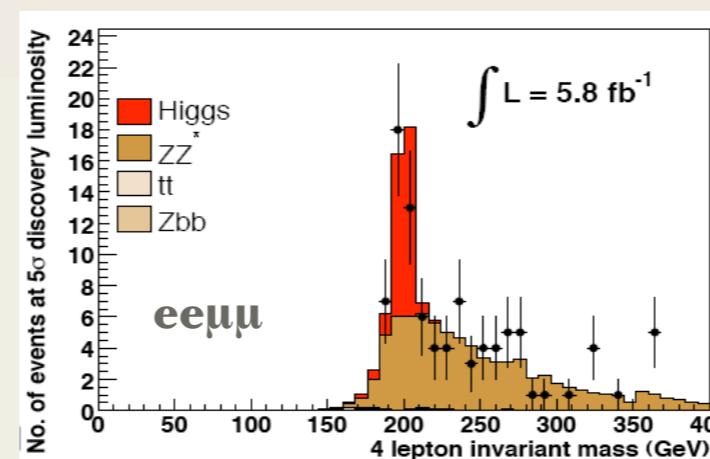
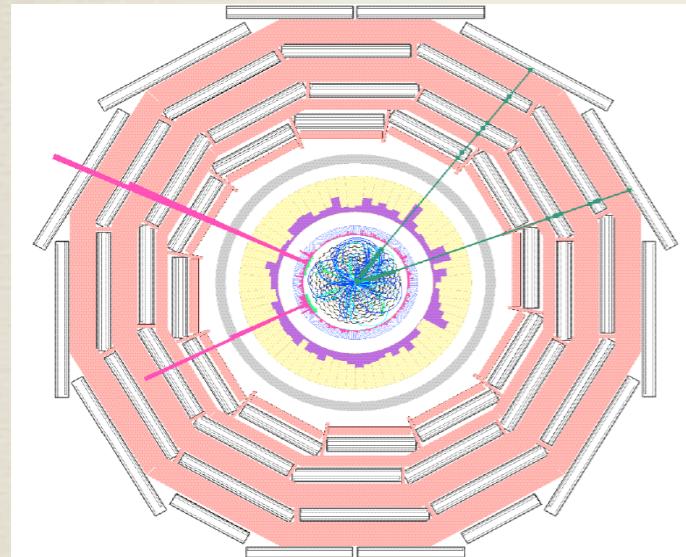


$h \rightarrow ZZ \rightarrow 4l$ assures discovery over entire high-mass range

$h \rightarrow WW \rightarrow llvv$ again important at LHC

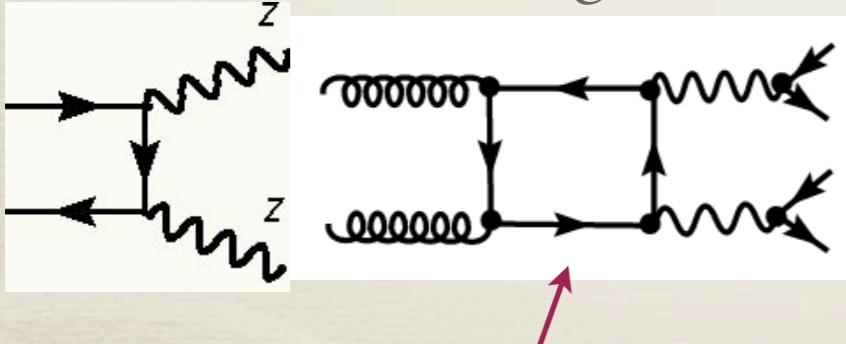
$h \rightarrow ZZ \rightarrow l_1 l_1 l_2 l_2$

- * Trigger: one $p_T > 20\text{-}25 \text{ GeV}$ or two $p_T > 10\text{-}15 \text{ GeV}$ leptons
 Reconstruct: at least one $Z \rightarrow ll$ decay

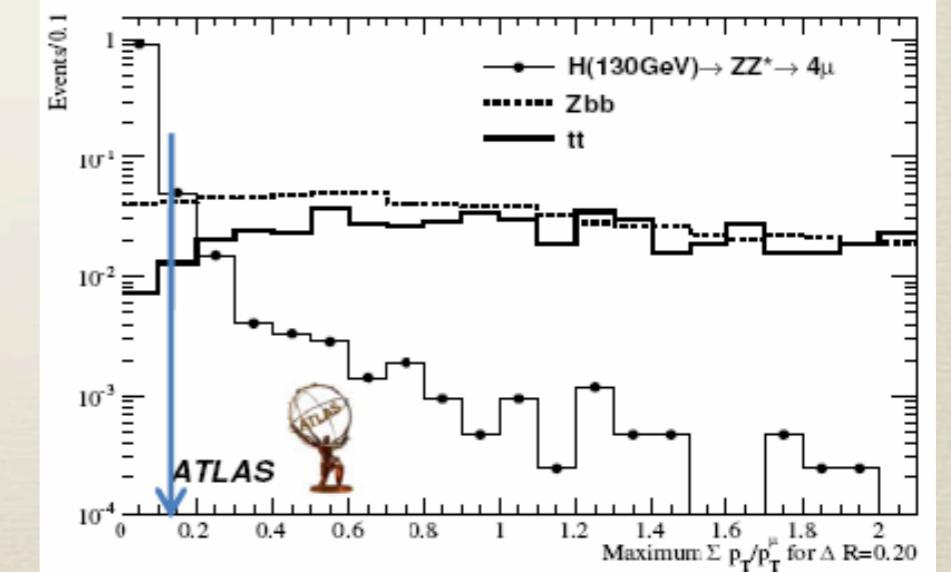
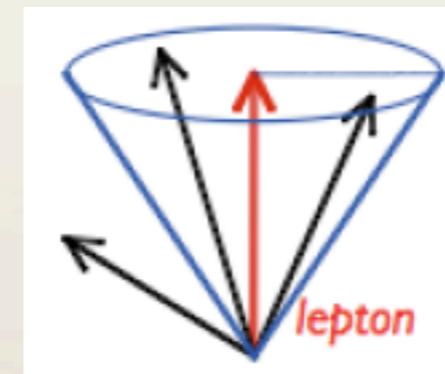


Reducible: $tt \rightarrow llvvbb$, Zbb with semi-leptonic b-decay

Irreducible backgrounds:



Formally NNLO, but enhanced by gg luminosity



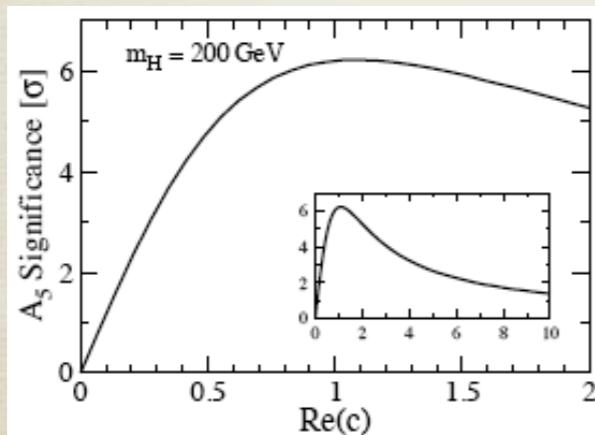
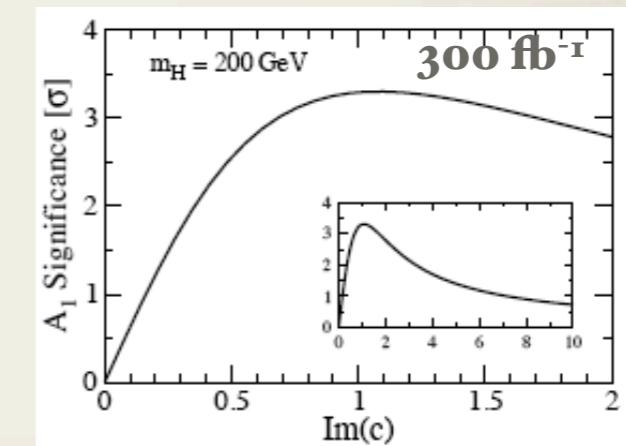
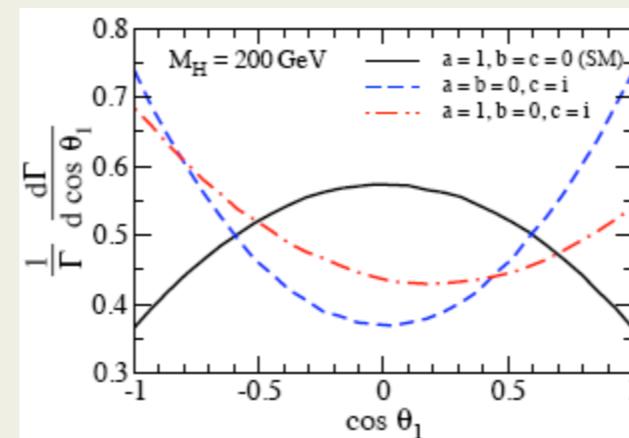
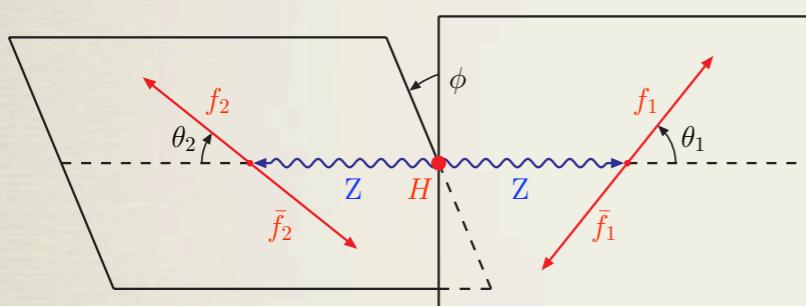
CP studies in $h \rightarrow ZZ$

- * Can use this channel to test for CP-even, odd properties

$$V_{hZZ} = \frac{2iM_Z^2}{v} \left\{ a g_{\mu\nu} + b \frac{p_\mu p_\nu}{M_Z^2} + c \epsilon_{\mu\nu\rho\sigma} \frac{p^\rho k^\sigma}{M_Z^2} \right\}$$

$(p_\mu = q_\mu^{V1} + q_\mu^{V2}, \quad k_\mu = q_\mu^{V1} - q_\mu^{V2})$

a,b: CP-even
c: CP-odd
SM: $a=1, b=c=0$



$$\mathcal{A}_1 = \frac{\Gamma(\cos \theta_1 > 0) - \Gamma(\cos \theta_1 < 0)}{\Gamma(\cos \theta_1 > 0) + \Gamma(\cos \theta_1 < 0)}$$

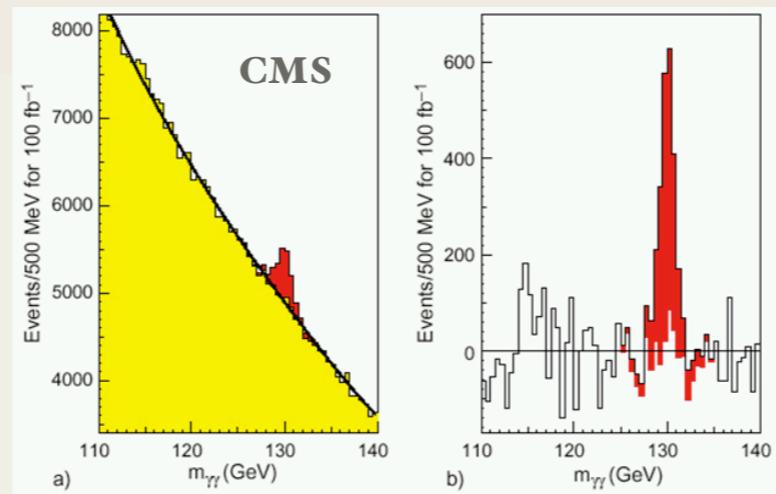
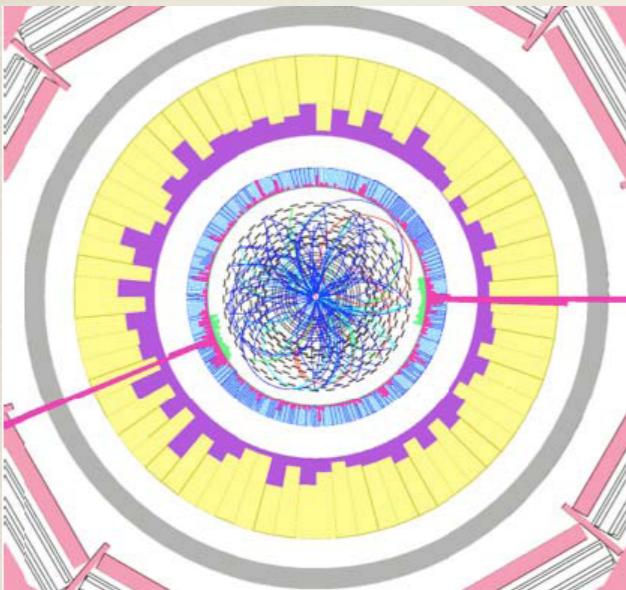
(either eeμμ, or llll with one pair reconstructing to Z)

Other asymmetries possible

$$O_5 = \sin \theta_1 \sin \theta_2 \sin \phi [\sin \theta_1 \sin \theta_2 \cos \phi - \cos \theta_1 \cos \theta_2]$$

$h \rightarrow \gamma\gamma$

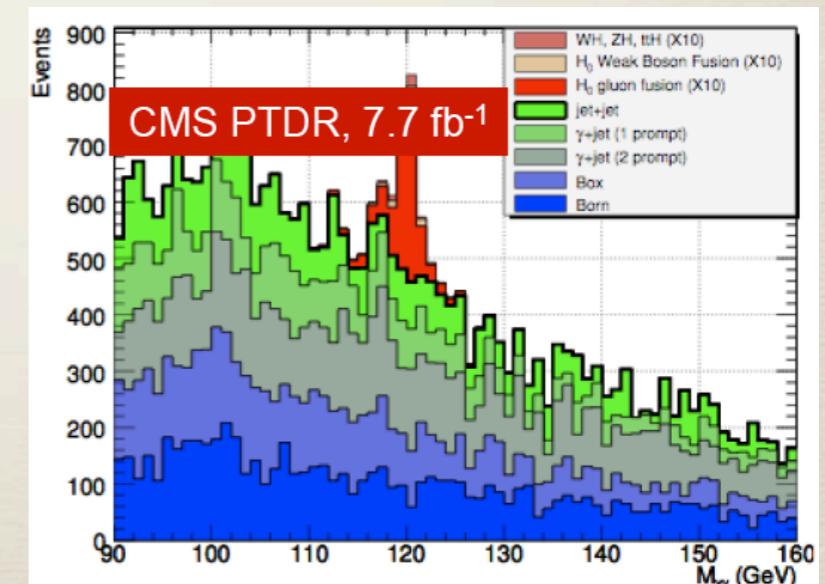
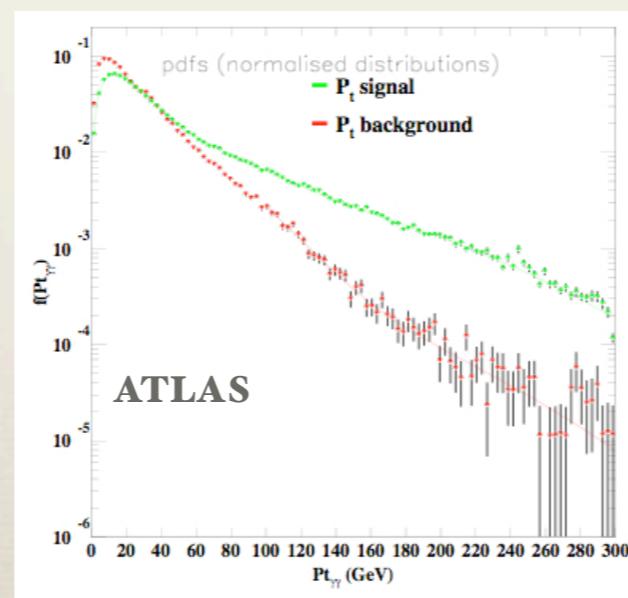
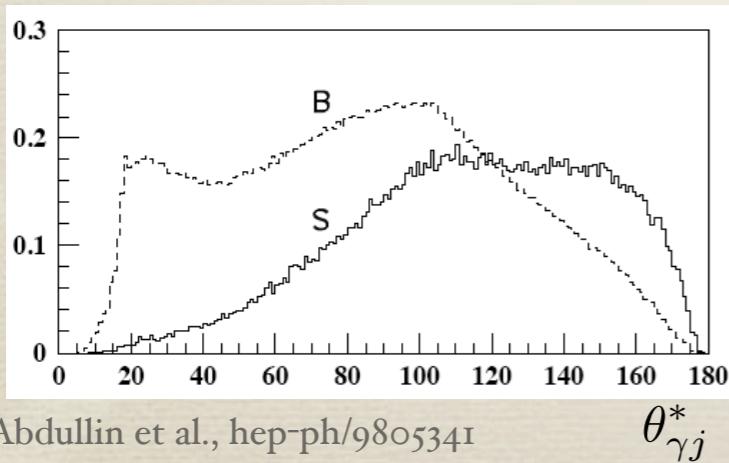
- * Trigger: 1-2 photons; Reconstruct: $p_T > 40$ GeV, $p_T > 25$ GeV
Excellent EM calorimeter resolution; calibrate with $Z \rightarrow ee$



Huge $\pi^0 \rightarrow \gamma\gamma$ background; measure from sideband

50: 20–30 fb⁻¹ for $M_H < 140$ GeV

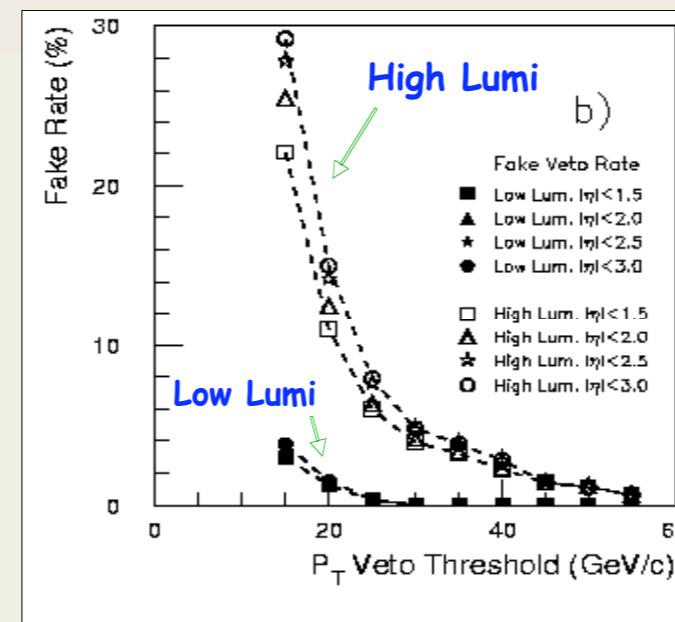
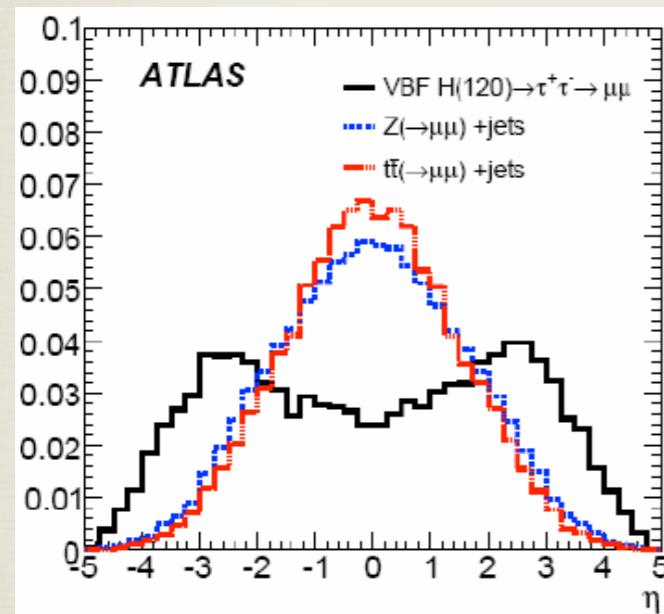
Additional handles with jets;
consider $\gamma\gamma + j, 2j$



Nisati, KITP '08

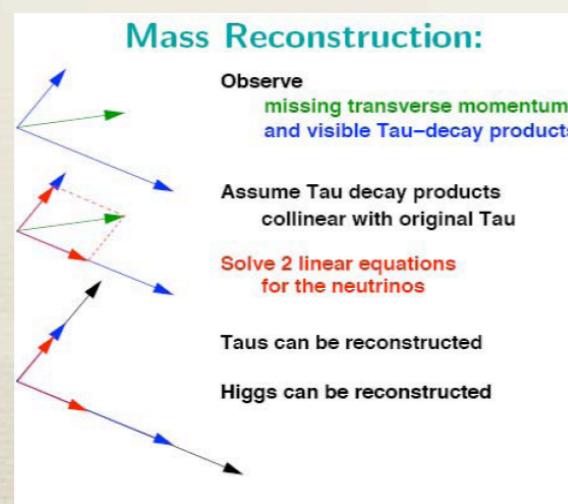
WBF $h \rightarrow \tau\tau$

- * Two tagging jets: $E_T > 40$ GeV, $\eta_{jj} > 4$, $M_{jj} > 500\text{--}1000$ GeV
Higgs decay products between tagging jets; central-jet veto
 $\tau\tau \rightarrow ll, lh$ modes possible

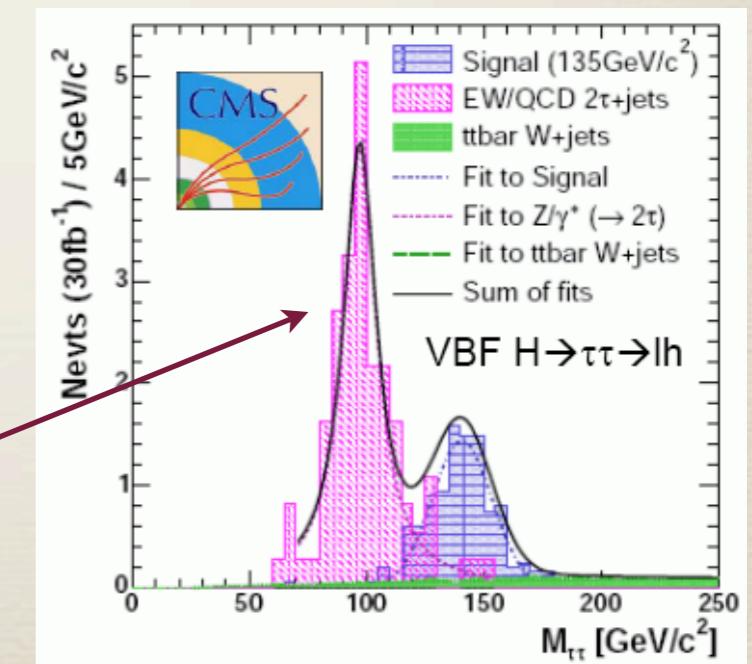


Pile-up introduces fake jets;
central-jet veto a concern at
high luminosities

Collinear approximation
for τ 's (highly boosted):



Data-driven
techniques to
control $Z \rightarrow \tau\tau$
line-shape

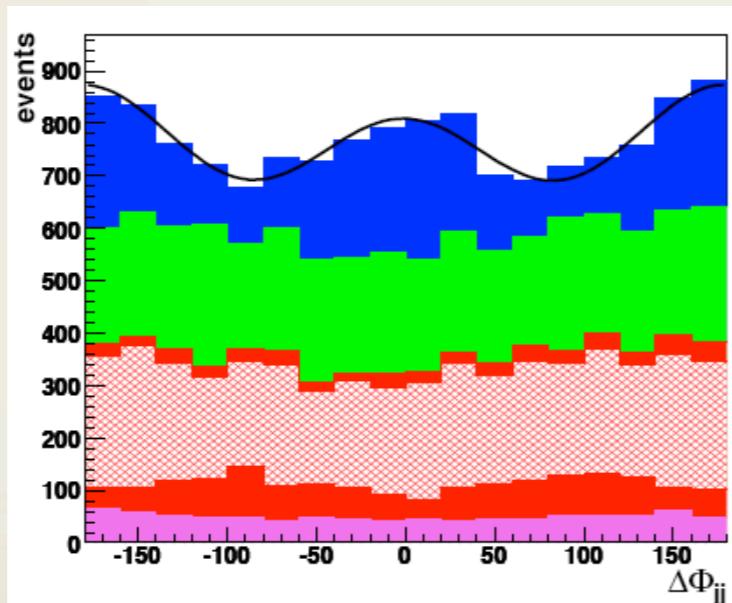


CP studies in h+2 jets

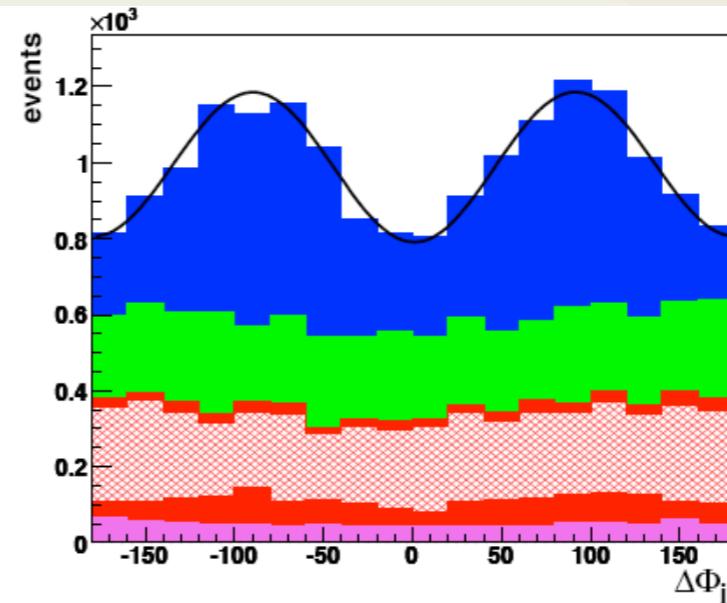
- * Use kinematics of h+2 jets to get at Higgs properties (both WBF and gluon-fusion h+2 jets); decay modes WW and $\tau\tau$

CP-even: $\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi} \frac{h}{v} G_{\mu\nu}^a G_a^{\mu\nu}$

CP-odd: $\mathcal{L}_{eff} = \frac{\alpha_s}{8\pi} \frac{a}{v} G_{\mu\nu}^a G_{\rho\sigma}^a \epsilon^{\mu\nu\rho\sigma}$ \Leftarrow comes from γ^5 fermion coupling in loop



CP-even
 $A = 0.100 \pm 0.039$
 $\Delta\Phi_{max} = 5.8 \pm 15.3$



h → WW
 $A = 0.199 \pm 0.034$
 $\Delta\Phi_{max} = 93.7 \pm 5.1$

Zeppenfeld, 2009
Zurich Higgs
workshop and refs
within

$$\epsilon_{\mu\nu\rho\sigma} b_+^\mu j_+^\nu b_-^\rho j_-^\sigma = 2 p_{T,+} p_{T,-} \sin(\phi_+ - \phi_-) = 2 p_{T,+} p_{T,-} \sin \Delta\phi_{jj}$$

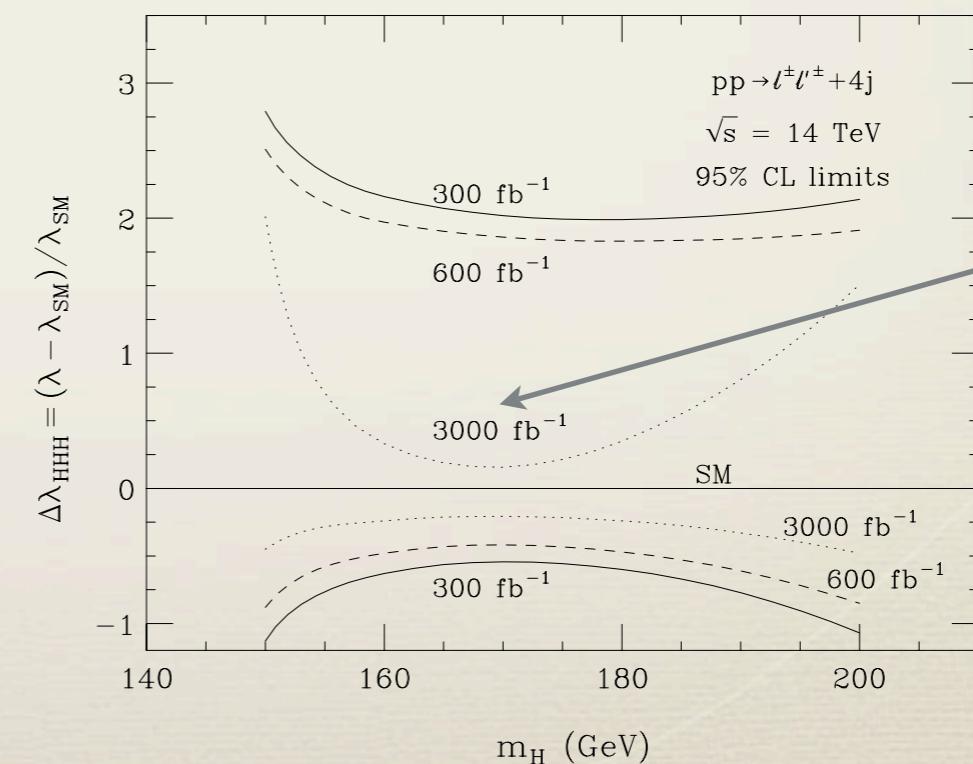
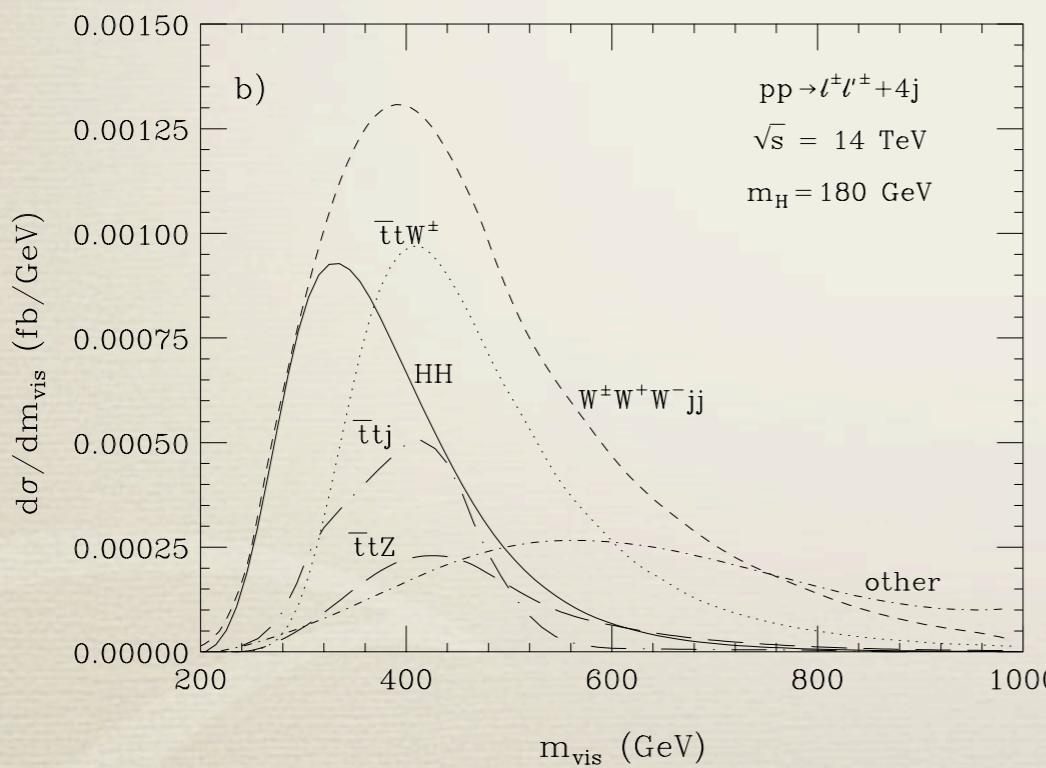
Measuring the Higgs potential

- * Form of SM Higgs potential makes definite predictions

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2$$

$$\Rightarrow g_{hhh} = 3M_H^2/v$$

Probe in $gg \rightarrow hh \rightarrow W^+W^-W^+W^- \rightarrow l^+l^- + 4j + \text{missing p}_T$
 (one possible final state)



super-LHC
 luminosity
 upgrade
 required

Global analysis of couplings

- * Observe Higgs in many modes: gluon-fusion, WBF, W/Z+h (not discovery at LHC, but after M_H known Butterworth et al., 0802.2470)

$$\sigma_p \times BR(h \rightarrow xx) = \underbrace{\left(\frac{\sigma_p}{\Gamma_p} \right)_{SM}}_{\text{NP effects cancel, calculate}} \times \underbrace{\frac{\Gamma_p \Gamma_x}{\Gamma}_{\text{measure}}}_{\text{NP effects cancel, calculate}}$$

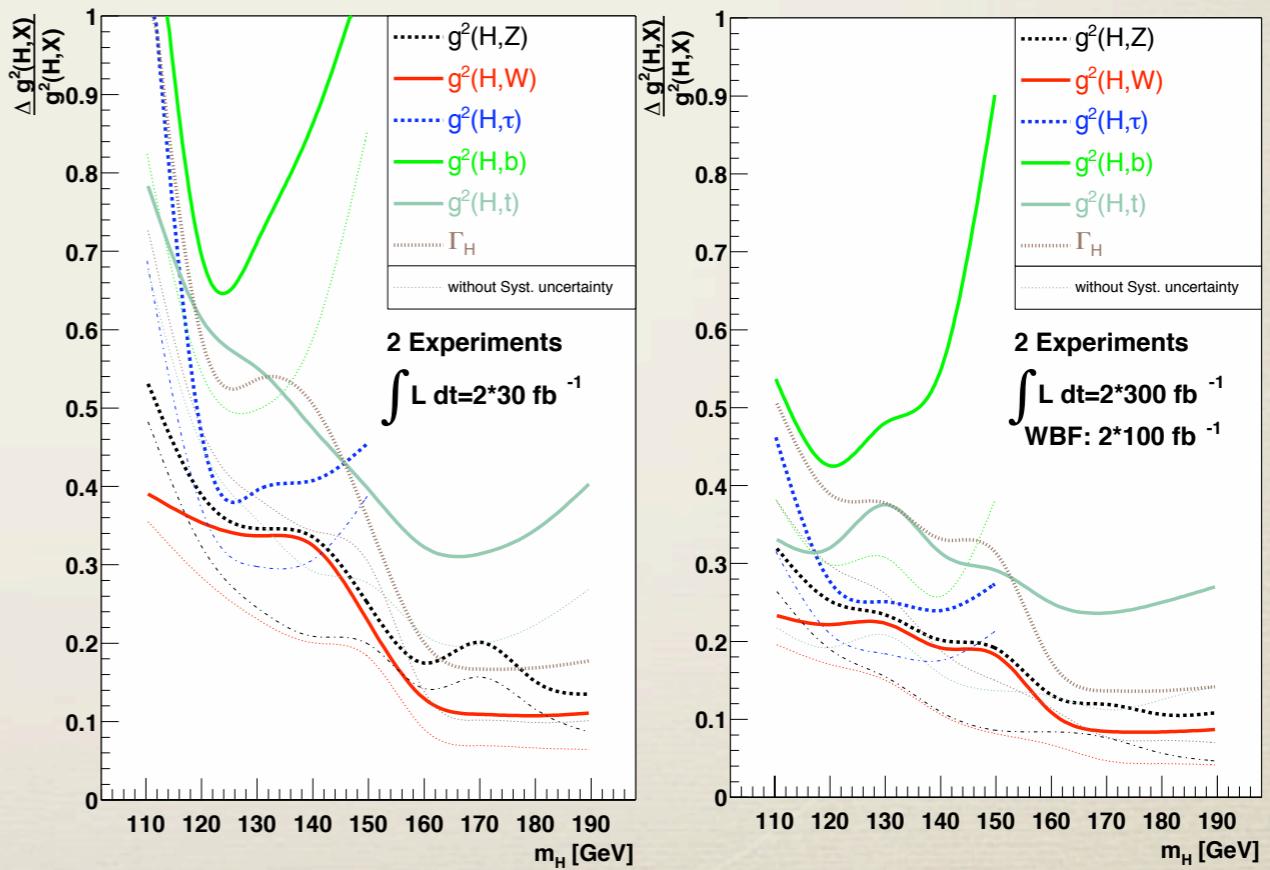
Scaling degeneracy if total width unknown:

$$\Gamma_i \rightarrow f \Gamma_i, \Gamma \rightarrow f^2 \Gamma$$

Mild assumption: $g_{hVV}^2 < 1.05 \times g_{hVV,SM}^2$

Allows any # of scalar doublets, new particles in loops, small contributions of scalar triplets

\Rightarrow Assumption+VBF measurement of $(\Gamma_V)^2/\Gamma$ breaks degeneracy

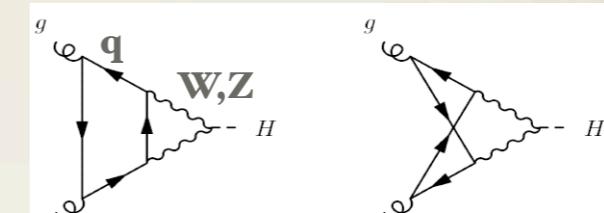
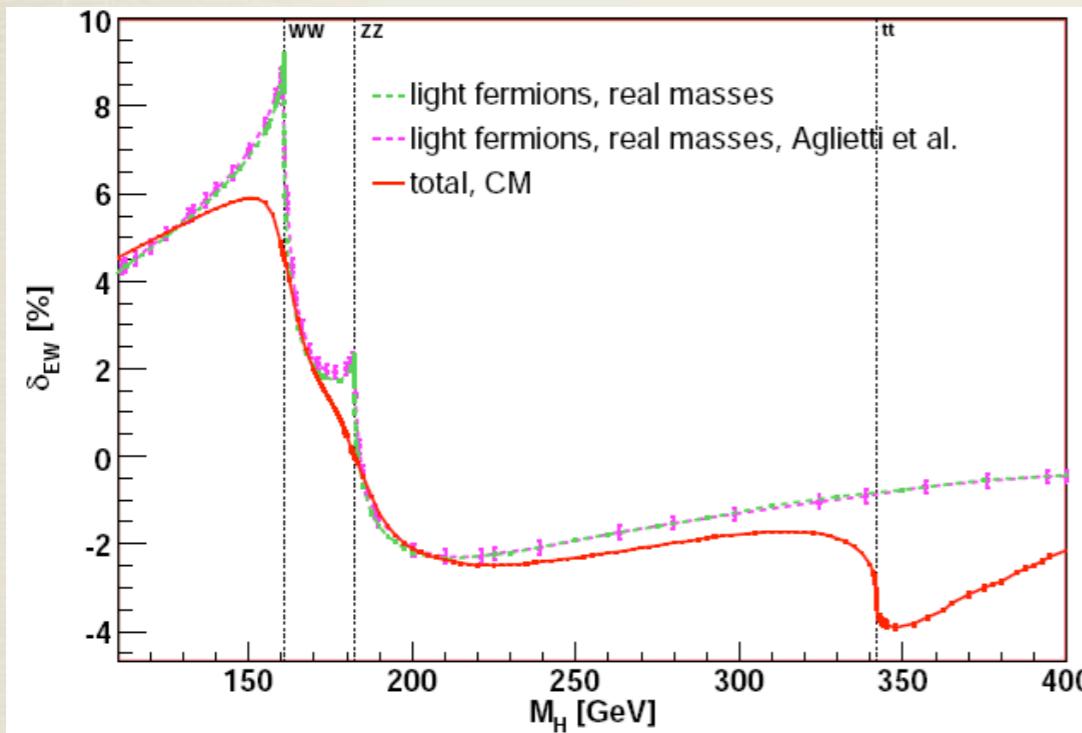


Precision Higgs physics: mixed QCD- EW corrections

Electroweak corrections

- * Significant EW corrections arising from light quarks

(Aglietti, Bonciani, Degrassi, Vicini hep-ph/0404071; Actis, Passarino, Sturm, Uccirati 0809.1301)



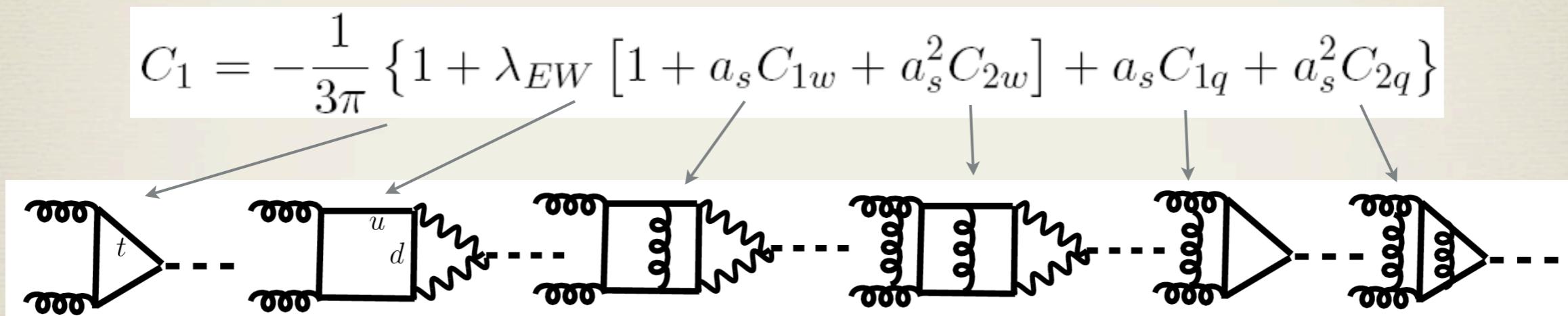
$$\sigma_{ew} = \sigma_0(1 + \delta_{ew})$$

What is their K-factor? 1 (no QCD correction, *partial factorization*), 3-4 (same as top-quark piece, *complete factorization*), ...?

- * Validity of top-quark EFT to 1 TeV inspires integrating out W,Z to compute QCD corrections; normalize to exact 2-loop result

EFT formulation

$$\mathcal{L} = -\alpha_s \frac{C_1}{4v} H G_{\mu\nu}^a G^{a\mu\nu}$$



Radius of convergence: $M_H \leq M_W$

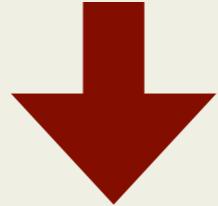
However, top-quark EFT valid to $1 \text{ TeV} > 2m_t$; reason to expect similar here

⇒ exact for dominant radiation pieces in resummation limit
 $\tau = M_H^2 / \hat{S} \rightarrow 1$ for all M_H

Factorization in EFT

$$\mathcal{L} = -\alpha_s \frac{C_1}{4v} H G_{\mu\nu}^a G^{a\mu\nu}$$

$$C_1 = -\frac{1}{3\pi} \left\{ 1 + \lambda_{EW} [1 + a_s C_{1w} + a_s^2 C_{2w}] + a_s C_{1q} + a_s^2 C_{2q} \right\}$$



$$C_1^{fac} = -\frac{1}{3\pi} (1 + \lambda_{EW}) \left\{ 1 + a_s C_{1q} + a_s^2 C_{2q} \right\}$$

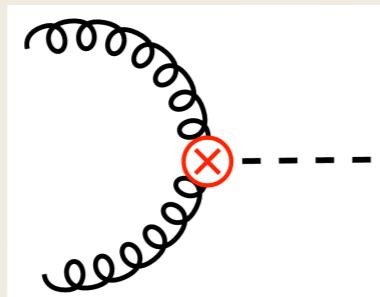
Factorization holds if $C_{1w}=C_{1q}$, $C_{2w}=C_{2q}$

$$C_{1q} = \frac{11}{4}, \quad C_{2q} = \frac{2777}{288} + \frac{19}{16} L_t + N_F \left(-\frac{67}{96} + \frac{1}{3} L_t \right)$$

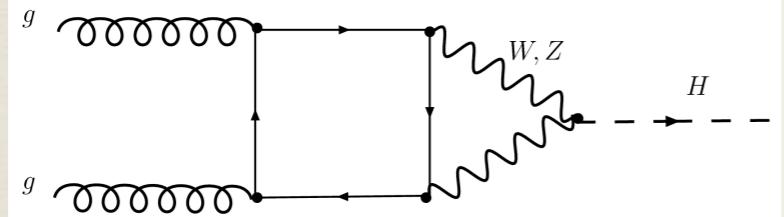
$$\lambda_{EW} = \frac{3\alpha}{16\pi s_W^2} \left\{ \frac{2}{c_W^2} \left[\frac{5}{4} - \frac{7}{3} s_W^2 + \frac{22}{9} s_W^4 \right] + 4 \right\}$$

Matching to the EFT I

Matching at $\mathcal{O}(\alpha)$:



$$= -\frac{1}{3\pi} \frac{\alpha_s}{v} \lambda_{EW} \mathcal{M}_0$$

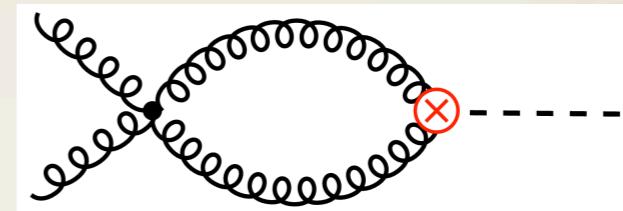
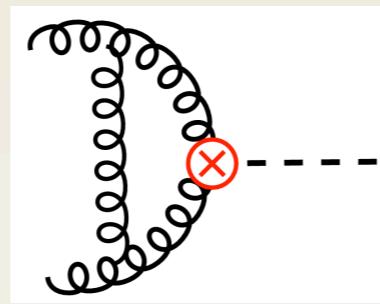
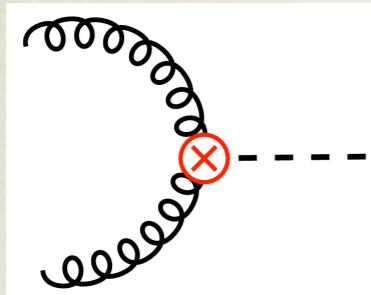


$$= \mathcal{A}^{(2)}(M_H^2 = 0) \mathcal{M}_0 + \mathcal{O}\left(\frac{M_H^2}{M_{W,Z}^2}\right)$$

⇒ Equate to get λ_{EW}

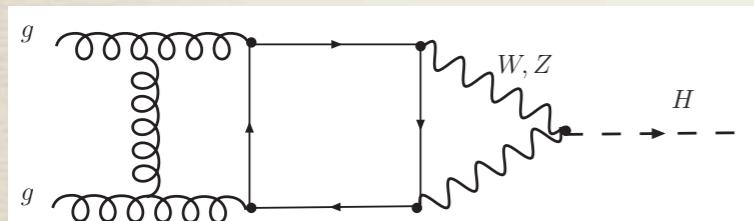
Matching to the EFT II

Matching at $\mathcal{O}(\alpha_s)$:

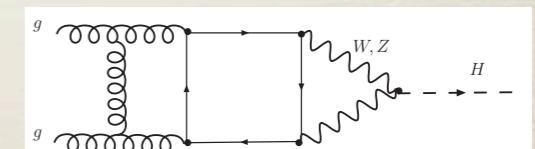
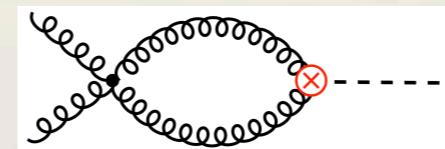
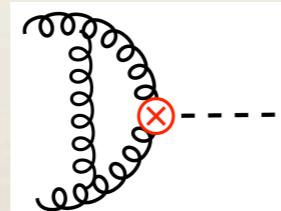
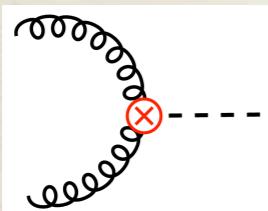


||

$$-\frac{1}{3\pi} \frac{\alpha_s}{v} \lambda_{EW} (\alpha_s C_{1w}) \mathcal{M}_0$$



$$= \mathcal{A}^{(3)}(M_H^2 = 0) \mathcal{M}_0 + \mathcal{O}\left(\frac{M_H^2}{M_{W,Z}^2}\right)$$

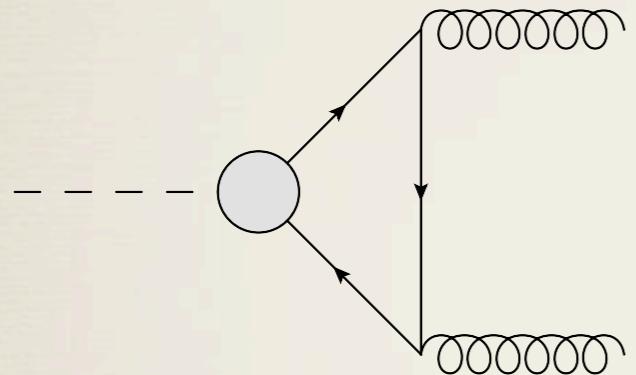


→ gives C_{IW}

EFT justification

Did we get all the needed operators? Only other same-order operator:

$$\frac{H}{v} \bar{q} \not{D} q$$



vanishes when inserted into EFT graphs

$$\mathcal{F}_\Gamma \sim \sum_\gamma \mathcal{F}_{\Gamma/\gamma} \circ T_{k,p_i} \mathcal{F}_\gamma$$

Reduced graphs:
only light lines,
quantum
corrections to
operators

Large-mass Feynman integral expansion: V. Smirnov

Subgraphs: contain all
massive props, Taylor
expand (EFT operators)

Check that all 0,1,2,3-loop subgraphs
contained in EFT or higher power ✓

Calculational procedure

Generate 3-loop diagrams for $g(p_1) + g(p_2) \rightarrow H(p_H)$

Taylor expand each diagram in M_H by applying:

$$\mathcal{D}\mathcal{F} = \sum_{n=0}^{\infty} (p_1 \cdot p_2)^n [D_n \mathcal{F}]_{p_1=p_2=0}$$

$$D_0 = 1, \quad D_1 = \frac{1}{d} \square_{12}, \quad D_2 = -\frac{1}{2(d-1)d(d+2)} \{ \square_{11} \square_{22} - d \square_{12}^2 \}$$

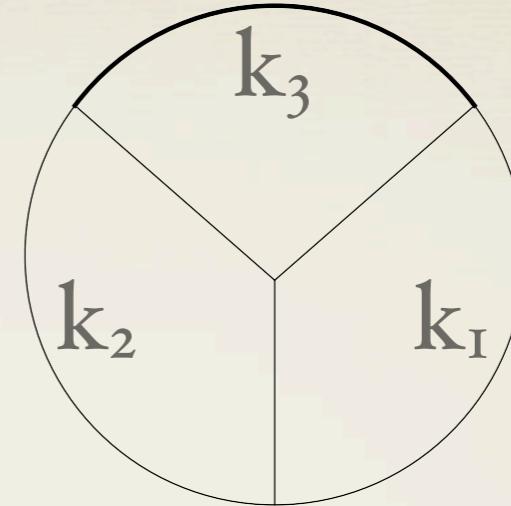
$$\sum \mathcal{F} = \mathcal{A} \left\{ g_{\mu\nu} - \frac{p_{2\mu}p_{1\nu}}{p_1 \cdot p_2} \right\} \delta^{ab} \epsilon_a^\mu(p_1) \epsilon_b^\nu(p_2) \equiv \mathcal{M}_{\mu\nu}^{ab} \epsilon_a^\mu(p_1) \epsilon_b^\nu(p_2)$$

$$\mathcal{A} = \frac{1}{8(d-2)} \left\{ g^{\mu\nu} - \frac{p_1^\mu p_2^\nu + p_2^\mu p_1^\nu}{p_1 \cdot p_2} \right\} \delta_{ab} \mathcal{M}_{\mu\nu}^{ab}$$

Leading term in \mathcal{A} gives C_{IW} upon comparison with LEFT; need through $n=2$

Structure of result

Coefficients in expansion are 3-loop vacuum bubbles:



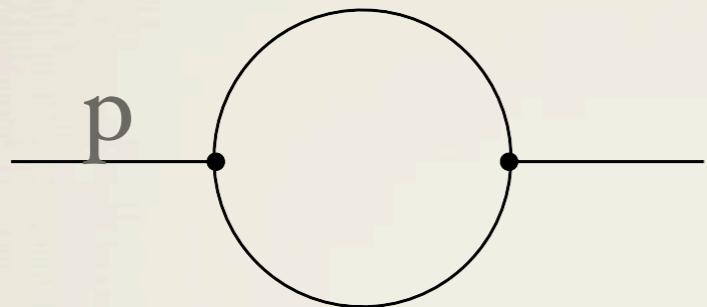
$$\begin{aligned}\mathcal{I}(\vec{\nu}_i) &= \int \prod_{j=1}^3 d^d k_j \frac{1}{k_1^{2\nu_1} k_2^{2\nu_2} (k_3^2 - M_{W,Z}^2)^{\nu_3} (k_1 - k_2)^{2\nu_4} (k_2 - k_3)^{2\nu_5} (k_3 - k_1)^{2\nu_6}} \\ &= \int \prod_{j=1}^3 d^d k_j \mathcal{D}\end{aligned}$$

Use integration-by-parts identities Chetyrkin, Tkachov '81;
translation invariance gives 9 eqs:

$$\int \prod_{j=1}^3 d^d k_j \partial_i [k_k \mathcal{D}] = 0$$

Integration-by-parts

In a simple case: 1-loop bubble diagrams



$$\mathcal{I}(\nu_1, \nu_2) = \int d^d k \frac{1}{k^{2\nu_1} (k + p)^{2\nu_2}}$$

Set

$$\int d^d k \frac{\partial}{\partial k^\mu} \left[\frac{k^\mu}{k^{2\nu_1} (k + p)^{2\nu_2}} \right] = 0$$

Derive

$$(d - 2\nu_1 - \nu_2)\mathcal{I}(\nu_1, \nu_2) - \nu_2\mathcal{I}(\nu_1 - 1, \nu_2 + 1) + \nu_2 p^2 \mathcal{I}(\nu_1, \nu_2 + 1) = 0$$

Apply to

$$\mathcal{I}(1, 1) \Rightarrow \mathcal{I}(1, 2) = -\frac{d - 3}{p^2} \mathcal{I}(1, 1)$$

Apply functional relation to progressively more complicated integrals; all in terms of $\mathcal{I}_{1,1}$

Integration-by-parts

Example of IBP equation for 3-loop calculation:

$$\{-\nu_4 \mathbf{1}\cdot\mathbf{4}^+ - \nu_6 \mathbf{1}\cdot\mathbf{6}^+ + \nu_4 \mathbf{2}\cdot\mathbf{4}^+ + \nu_6 \mathbf{3}\cdot\mathbf{6}^+ + \nu_6 \mathbf{6}^+ + (d-2\nu_1 - \nu_4 - \nu_6)\}$$



$$I(\nu_1, \nu_2, \nu_3, \nu_4, \nu_5, \nu_6) = 0$$

Operators acting on the
arguments of I

Apply IBP eqs to list of seed integrals:
 $I(I,O,I,I,I,O), I(I,O,I,2,-I,I), \dots$

Solve resulting system of equations Laporta ‘or

>1000000 seeds; express in terms of 2 master
integrals: $I(I,O,I,I,I,O)$ and $I(I,I,I,O,I,I)$

Some examples

$$\begin{aligned}\mathcal{I}(1, 1, 1, 1, 1, 1) &= \frac{2(3d-8)(3d-10)}{(d-4)^2} \mathcal{I}(1, 0, 1, 1, 1, 0) - \frac{2(d-3)}{d-4} \mathcal{I}(1, 1, 1, 0, 1, 1) \\ \mathcal{I}(1, -1, 1, 1, 1, 1) &= \frac{d-2}{d-4} \mathcal{I}(1, 0, 1, 1, 1, 0) \\ \mathcal{I}(1, 1, 1, 1, 2, 1) &= -\frac{3(3d-8)(3d-10)(d-5)}{(d-6)(d-4)} \mathcal{I}(1, 0, 1, 1, 1, 0) + (2d-6) \mathcal{I}(1, 1, 1, 0, 1, 1) \\ \mathcal{I}(1, -2, 1, 1, 1, 3) &= \frac{d(d-2)(3d-8)}{(d-8)(d-6)(d-4)} \mathcal{I}(1, 0, 1, 1, 1, 0) \\ \mathcal{I}(1, 1, 3, 1, 2, 3) &= \frac{9}{16} \frac{(3d-14)(3d-20)(3d-10)(3d-16)(3d-8)(d-7)}{(d-8)(d-10)} \mathcal{I}(1, 0, 1, 1, 1, 0) \\ &\quad + \frac{3}{8} (3d-20)(d-3)(d-4)(d-5)(d-6) \mathcal{I}(1, 1, 1, 0, 1, 1)\end{aligned}$$

Can evaluate master integrals via simple Gamma functions

Analytical result

No renormalization needed (finite renormalization needed for top quark case)

$C_{IW}=7/6$, compared to factorization hypothesis $C_{IW}=C_{Iq}=11/4$

$(C_{Iq}-C_{IW})/C_{Iq} \approx 0.6 \Rightarrow O(I)$ violation of assumption

Numerical effect on hadronic cross section?

Numerical test of CF

QCD corrections in EFT

$$\hat{\sigma}_{ij} = \sigma^{(0)} G_{ij}(z; \alpha_s)$$

$$G_{ij}(z; \alpha_s) = \sum_{n=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n G_{ij}^{(n)}(z)$$

$$G_{ij}^{(0)}(z) = \delta_{ig} \delta_{jq} \delta(1-z)$$

partial factorization



$$\sigma_{EW}^{LO} = \sigma_{t,lf}^{(0)} G_{ij}^{(0)}(z) ,$$

$$\sigma_{EW}^{NLO} = \sigma_{t,lf}^{(0)} \left\{ G_{ij}^{(0)}(z) [1 + a_s(C_{1w} - C_{1q})] + a_s G_{ij}^{(1)}(z) \right\} ,$$

actual result



$$\sigma_{EW}^{NNLO} = \sigma_{t,lf}^{(0)} \left\{ G_{ij}^{(0)}(z) [1 + a_s(C_{1w} - C_{1q}) + a_s^2 (C_{2w} - C_{2q} + C_{1q}(C_{1q} - C_{1w}))] \right.$$

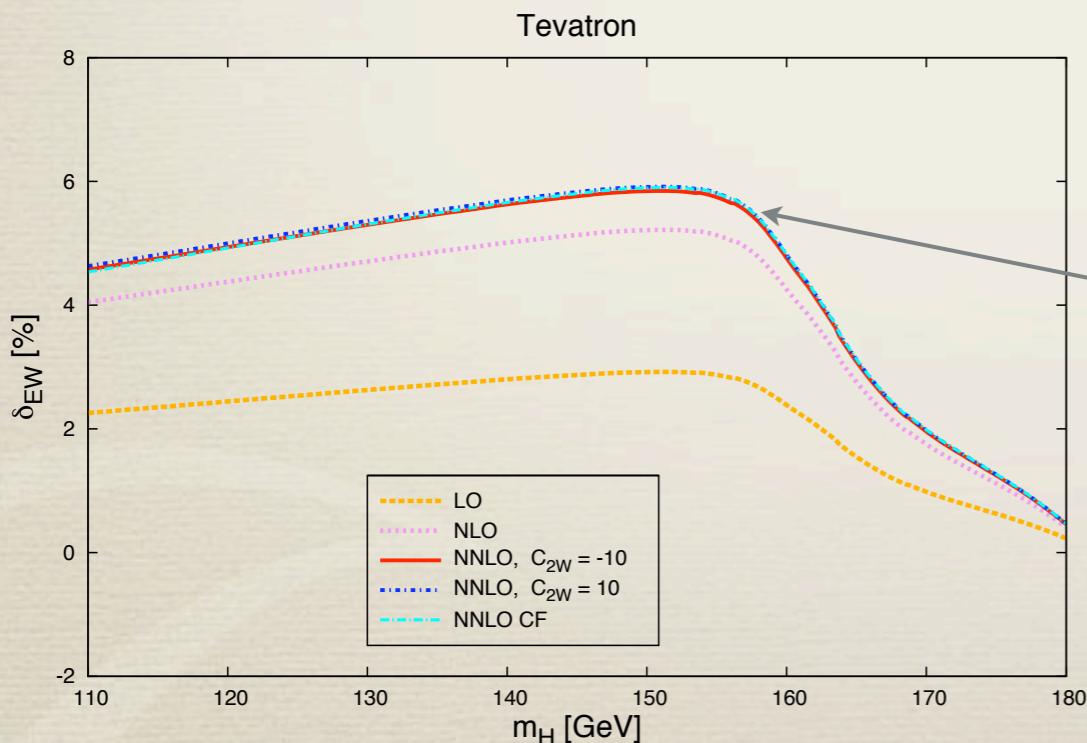
$$\left. + a_s G_{ij}^{(1)}(z) [1 + a_s(C_{1w} - C_{1q})] + a_s^2 G_{ij}^{(2)} \right\} ,$$

complete factorization



$$\sigma_{EW}^{NNLO\ CF} = \sigma_{t,lf}^{(0)} G_{ij}(z; \alpha_s) ,$$

Full mass-dependent 2-loop EW corrections
of Actis et al. 0809.1301



Difference between CF and actual:
 $a_s(C_{1w} - C_{1q})$

Small compared to $a_s G^{(1)}(z)$

Conclusions

- * We must find a Higgs boson or something else which consistently breaks EW symmetry
- * Phenomenology of Higgs intricate and highly dependent on its mass; detailed experimental program needed to find it
- * Very sensitive to quantum effects; better have a good handle on QCD!
- * Tevatron beginning to reach into allowed SM mass region; LHC will pick up the search later this year
- * Rich program to determine whether the Higgs is SM or not with LHC measurements